



## Introduction to the special issue on stability under finite deformation

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# Introduction to Special Issue on Stability under Finite Deformation

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In the botanical gardens of Grenoble, France, there is small bridge, just a few meters long, made of *pre-stressed* concrete. The bridge dates back to 1855, and is thought to be the oldest manufactured prestressed structure made with concrete. In general, a pre-stressed bridge span is reinforced by parallel metallic rods embedded below the horizontal mid-plane and put under tension, so that the slab is bent slightly upwards. When the bridge is put in place, the span is subjected to its own weight and to external loads, which ensure that the concrete is compressed everywhere. Without the prestress, the weight and loads would bend the span downwards, and create a zone of tension, of which concrete cannot sustain much. This technology thus combines the compressive strength of concrete with the tensile strength of the metallic rods. We can say that the resulting structure is *stabilized* by pre-stress.

Of course, a pre-stress or a pre-strain can conversely lead to the *destabilization* of a solid. A classic example is that of the twisting instability of a closed ring: take an elastic straight rod, bend it into a circle by bringing the two ends together, but give the rod a twist before gluing its ends. For a sufficiently high twist, the elastic ring is unstable and folds unto itself, by forming an eight-shape curve (with about two turns of pre-twist) or more elaborate shapes (for higher twists).

Stability studies are highly important for many engineering materials, be they subjected to small pre-strains such as those imposed on pre-stressed concrete, as

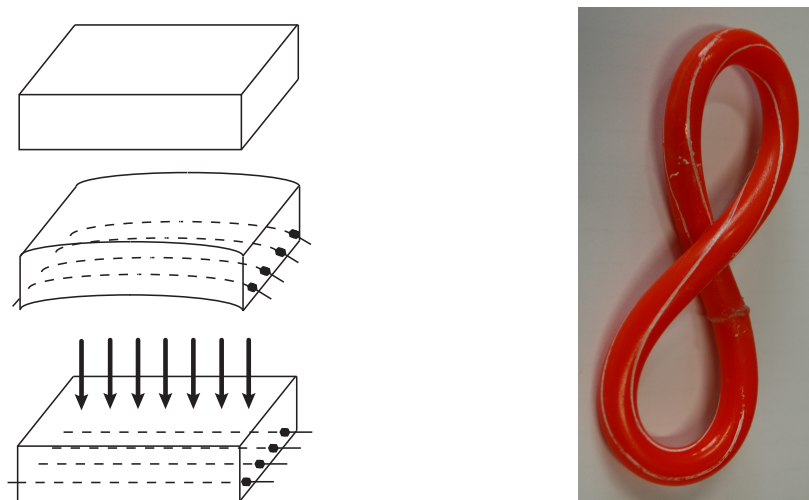


Figure 1: On the left: Stabilizing a bridge span made of concrete by reinforcing it with steel rods put under tension. Once the span is put in place and subject to loads, the concrete is compressed everywhere. On the right: Destabilizing a ring by twisting a polyurethane rod before gluing its two ends together. For a large enough pre-twist, the ring folds unto itself in a eight-shape.

well as to finite pre-strains, such as those imposed on elastomers in bridge bearings and engine mountings. These studies have developed into a fundamental topic of mathematical modeling, because they are relevant not only to engineering problems, but also to many biological, biomedical, and biomechanical applications (DNA mechanics, cell stiffness, cellular structures, plant growth, microbial filaments, deformation of arteries and veins, skin wrinkling, etc.)

The discipline of elastic stability has a long and distinguished history, dating back at least to the works of Euler. For a comprehensive bibliography and an extensive treatment of most known problems, we refer for example to the textbook by (Bažant & Cedolin 2003). For this Introduction to the Special Issue on *Stability under Finite Deformations*, we chose to evoke three seminal papers which we believe qualify as *pioneering* works in this field, and yet seem to be little-known and forgotten (to the best of our knowledge). First, an 1829 article by Augustin-Louis (Cauchy 1829), which contains the first derivation of the equations of motion in a solid which is already in a state of stress. According to (Truesdell 1966), his ‘results were not understood and were reported obscurely or even incorrectly by nineteenth century expositors’. By the time (Lord Rayleigh 1906) modeled the Earth as a solid with initial stress, ‘Cauchy’s results had long been forgotten’, according to (Man & Lu 1987). Second, a most elegant four-page treatment of

the twisted elastic ring instability by John Henry (Michell 1889). According to (Goriely 2006), this Australian applied mathematician cuts an ‘almost tragic figure’, whose work has only recently begun to be fully appreciated. In particular, (Goriely 2006) found out that his result on ring instability has been rediscovered at least three times in the 20th century. Finally, a 1940 article by the Belgian applied mathematician, Maurice Biot (1905-1985). Although Biot is well remembered for laying down the foundations of the theory of poro-elasticity, it is not so well acknowledged that he can be credited with the modern derivation of the incremental equations of non-linear elasticity (his results are beautifully synthesized in his 1965 monograph, now out-of-print). In his 1940 article in the *Journal of Applied Physics*, he gave a simple and direct exposition of the incremental equations of elastodynamics, and applied to the study of the influence of initial stress on seismic waves...

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