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Evolution of Cooperation in N-player Social Dilemmas: The Importance of being Mobile

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Abstract: *This paper addresses issues regarding the emergence of cooperation in evolutionary, spatial game-theoretic simulations. In the model considered, agents participate in a social dilemma with their neighbours and have the ability to move in response to environmental stimuli. Both the movement strategies and the game strategies (whether to cooperate or not) are evolved. In particular, we present results that compare the outcomes using the classical two player prisoner’s dilemma and a generalised N-player prisoner’s dilemma. We also explore the effect that agent density (the number of agents present per cell in the world) has on the evolution of cooperation in the environment. Finally, we discuss the movement strategies that are evolved for both cooperative and non-cooperative strategies.*

1 Introduction

Questions relating to cooperation and its emergence have been studied in a range of domains including economics, psychology, theoretical biology, and computer science. Researchers have explored the conditions necessary for cooperation to emerge among groups or societies of self-interested agents. Social dilemma games, such as the Prisoner’s Dilemma [4], have been adopted as a succinct representation of the conflict between individually selfish behaviours and collectively rational behaviours. Evolutionary game theory has been studied since the 1980s when ideas from evolutionary theory were incorporated into game theory [16].

A variety of social dilemmas have been studied with the majority of attention afforded to the 2-player prisoner’s dilemma. Many variations of this game exist, which allow researchers to explore questions regarding cooperation in the presence of noise, trust, spatial mechanisms and other extensions. One interesting extension that has been explored in the literature is that of N-player social

dilemmas [31] where N agents participate simultaneously in the interaction. Each agent can cooperate or defect, and receives a reward based on the number of cooperators present. Additionally, cooperators incur a cost to interact while defectors do not.

In this work, we consider populations of agents participating in both the 2-player and N -player versions of the prisoner’s dilemma. We adopt a spatial model where agents’ interactions are defined by some topological constraints. Much recent work has focused on the effect of such constraints [27, 20, 15]. We use a toroidal lattice where agents may interact with their immediate eight neighbours, if any. We further imbue the agents with the ability to move based on environmental stimuli. The role of mobility in the evolution of cooperation has grown in importance and recognition in recent decades with several researchers demonstrating its use in the promotion of cooperation in artificial life simulations [1, 29]. We adopt an evolutionary framework where successive populations are evolved; the strategy for interacting in the games and the mobility strategy are both subject to evolution.

The N -player prisoner’s dilemma has not been widely studied in evolutionary models where agents are spatially situated with the inclusion of mobility. We wish to explore if any significant differences are prevalent between 2-player and N -player dilemmas in this context. Furthermore, we wish to examine the effect of varying the density of the agents in the environment, and finally we wish to analyse the movement strategies evolved in these conditions.

In this paper, we show through simulation that there is in fact a substantial difference between the 2-player and the N -player scenarios in terms of the likelihood of cooperation emerging for varying density levels. We demonstrate that for a range of density levels, cooperation emerges in the N -player case.

The paper outline is as follows: the next section discusses some related work in the field, section 3 outlines our model and approach, and section 4 presents and discusses our results. Finally conclusions and some potential future directions are presented.

2 Related Work

In this section we review some of the relevant research in the literature; we introduce some concepts pertaining to social dilemmas and discuss some work on spatial and evolutionary game theory and the role of mobility.

2.1 Social Dilemma Games

Social dilemma games (most famously the prisoner’s dilemma and its variants) have been studied in a wide range of domains due to their usefulness in capturing the conflict between individual and collectively rational behaviours. The prisoner’s dilemma in the classical game is described as follows: two players make a choice simultaneously to either cooperate or defect. Mutual cooperation yields a reward R for both participants. However, unilateral defection results

in a greater payoff, T , for the defector and a worse payoff, S , for the cooperator (the sucker's payoff). If both defect, both receive P as a payoff such that: $T > R > P > S$.

It has been argued that the N-player variant captures a wider set of dilemmas (e.g. donating to charity organisations, environmental issues etc.). In the N-player dilemma game there are N participants, and again, each player is confronted with a choice: to either cooperate or defect. In one formalism of the game [5], all players receive a benefit based on the number of cooperators present. Cooperators have to pay a cost. No such cost is borne by defecting players. For instance, let B represent some fixed benefit, N the number of players, c the cost and i the number of cooperators. Participants receive $(B \times i)/N$. Cooperators must pay c and thus receive a net reward of $((B \times i)/N) - c$. This, or similar, formulas have been adopted in several other works [21, 31, 26].

We represent the payoff obtained by a strategy which defects given i cooperators as $D(i)$ and the payoff obtained by a cooperative strategy given i cooperators as $C(i)$. Defection represents a dominant strategy, that is, for any individual, moving from cooperation to defection is beneficial for that player in that they still receive a benefit without the cost:

$$D(i) > C(i) \quad 0 < i \leq N - 1$$

However, if all participants adopted this dominant strategy, the resulting scenario would be a sub-optimal, and from a group point of view, irrational outcome:

$$C(N) > D(0)$$

If any player changes from defection to cooperation, the society performs better:

$$(i + 1)C(i + 1) + (N - i - 1)D(i + 1) > (i)C(i) + (N - i)D(i)$$

In multi-person games, the problem of avoiding exploitation, or free riders, is more difficult, and cooperation may be harder to achieve. In 2-player games, reciprocity has been explored as a means to engender cooperation [18]. However, in N-person games reciprocity may be less advantageous. In order for an agent to punish a defector by defecting in retaliation, the agent must also punish all those that did cooperate.

2.2 Evolutionary N-player Games

There have been several other notable approaches to exploring the N-player prisoner's dilemma using the tools and approaches in evolutionary game theory. Yao and Darwen [31] explore the effect of group size in the evolution of cooperation. Strategies are represented using a generalised form of the representation employed by Axelrod and Dion[3]. In their experiments, it is shown that cooperation can be evolved in groups but that it becomes more difficult with increasing group size.

The effects of spatial influences on the evolution of cooperation among strategies participating in the N-players prisoner’s dilemma is explored by Suzuki and Arita [26]. The two spatial factors under investigation are on the *scale of interaction* (determines which neighbours to play with) and *scale of influence* (specifies which neighbouring candidates to choose for offspring). Results for simulations involving a *tit-for-tat* like strategy showed that cooperation becomes most wide-spread for a modest value of scale of interaction and that, as the cost of cooperation increases, the levels of cooperation decrease and a higher value of the scale of interaction is found. Results also indicate that higher cooperation levels are achieved for higher values of the *scale of influence*.

2.3 Mobility

Traditional spatial models promote the evolution of cooperation by constraining agent interactions to a particular static topology. Previous work has investigated structures such as lattices [19], small-world graphs [23], and scale-free graphs [22]. However, the inclusion of movement creates a more realistic model by allowing agents to respond to their current neighbourhood by moving within their environment.

Mobility is a form of network reciprocity [18], which has gone from being perceived as a hindrance to the emergence of cooperation to a key concept in its promotion. While unrestrained movement can, and does, lead to the ‘free-rider’ effect [7], allowing highly mobile defectors to go unpunished, using simple strategy rules [1, 12] or using mobility rates [17, 29] significantly curb the free-rider phenomenon allowing self-preserving cooperator clusters to form, and cooperation to proliferate.

Several mechanisms for the emergence of cooperation exist, but all essentially express a need for cooperators to either avoid interactions with defectors or increase and sustain interactions with other cooperators. Research in this domain is largely divided into two categories based on authors’ definition of mobility; all movement should be random [29, 17, 24, 2], or should be purposeful or strategically driven, but may indeed contain random elements [1, 10, 11, 13, 30, 28]. Random mobility can be used to describe the minimal conditions for the evolution of cooperation. Alternatively, contingent mobility has the capacity to be proactive. This is where individuals deliberately seek better neighbourhoods, rather than simply reacting to stimuli and randomly relocating.

The majority of the contingent mobility strategies in the literature are hand crafted or guided by heuristics. However, there has been some research [14, 8, 9] using evolutionary models to evolve movement strategies that are conducive to the emergence of cooperation. Ichinose et al. [12] also use an evolutionary model and investigates the coevolution of migration and cooperation. Agents play an N-player Prisoner’s Dilemma game after which they move locally according to an evolved probability vector. All agents are evolved to collectively follow or chase cooperators. The authors highlight the importance of flexibility in the direction of migration for the evolution of cooperation.

Chiong et al. [6] describe a random mobility model where a population

of agents interact in an N-player Prisoner’s Dilemma set in a fully occupied regular lattice. Pairs of agents move by exchanging grid positions. Mobility in this environment is a probability function based on the time an agent has spent in a location, and the relative fitness of the agent at the destination. The agents have a limited memory of past interactions, and past cooperator and defector levels. Cooperation is shown to be promoted under a limited small set of parameters including the cost to benefit ratio of cooperation and the movement radius.

Most recently, Suarez et al. [25] present a contingent mobility model, using the N-Player game, in which agents move toward locations with higher potential payoff. While cooperation does emerge, the authors do not elaborate on the specific effects of mobility, focusing more on the impact of the neighbourhood size.

3 Methodology

3.1 Environment & Agent Representation

The population of agents A inhabits a toroidal shaped diluted lattice with $L \times L$ cells, each of which can be occupied by up to one agent. The interaction and movement radii of agents is determined using the Moore neighbourhood of radius one. This comprises the eight cells surrounding an individual in a cell on the lattice. The agents can only perceive and play with those within this limited radius.

Each agent is represented by a genotype, which determines their strategy to interact with other agents and to move in the environment. The first section of the gene describes their strategy for playing the game: that is to cooperate or defect and the remaining sections determine how an agent will move. The remainder of the genotype encodes actions for a range of scenarios that may arise within the environment, including: encountering a cooperator, encountering a defector, or encountering both at once. If an agent meets a cooperator, they have a set of potential actions. These actions are as follows: remain where they are, move randomly, follow the cooperator or flee from it. Similarly these potential actions are mirrored when an agent meets a defector. The final section is used to determine actions when an agent meets both a defector and a cooperator. The actions are: flee from both cooperator and defector; follow both cooperator and defector; follow the cooperator and flee from the defector and the converse action (flee from the cooperator and follow the defector). During a simulation run, each potential action of an agent is determined by its genotype.

At each time step, agents participate in a single round of the Prisoner’s Dilemma with each of their neighbours, if any. The strategy with which agents play is fixed; either always cooperate or always defect. We choose to implement pure strategies in order to reduce the strategy space allowing us to more clearly examine the effect of mobility in these experiments. Agents are aware of the actions taken by their neighbours in a single round, but these memories do

not persist. Following this interaction phase, agents have the opportunity to take one step into an adjacent free cell according to their movement strategy. Movement will not occur if there is no adjacent free space, or if their strategy dictates that they remain in their current location. Isolated agents will take one step in a random direction.

3.2 Evolutionary Dynamics

The movement strategies adopted by the population are explored by using an ALife inspired evolutionary model. In a single generation, agents accumulate their payoffs received from playing the Prisoner’s Dilemma with their neighbours. This is used as a measure of fitness, and at the end of each generation, the agents are ranked according to this score. The bottom 20% are replaced with copies of the top 20%. This replacement strategy was chosen as it has been previously shown to produce a fair sampling of the population’s fitness while still allowing for convergence in a reasonable time frame. No other genetic operators are utilized. These offspring are randomly placed on the grid, and the other agents remain in the same place, thus maintaining any spatial clustering between generations. Following reproduction, the fitness score of the whole population is reset and a new generation begins.

3.3 Interaction model

In keeping with previous work, we adopt a well known formalism for the N-player prisoner’s dilemma. Letting B be a constant representing social benefit, c be the cost of cooperation and i the number of cooperators from a group of N agents, the following payoffs are used:

$$C(i) = \frac{B \times i}{N} - c$$

$$D(i) = \frac{B \times i}{N}$$

The following constraints hold: $B > c$ and both B and c are positive values.

Considering the N-player dilemma, when $N=2$ and attempting to align with the classical interpretation of the 2-player prisoner’s dilemma, we also require that $B < 2c$. Values chosen in this research that are in keeping with previous studies in the field are: $B = 5, c = 3$.

For example, in mapping this back to the two player games, we use the following payoff matrix:

Table 1: Prisoner’s Dilemma game matrix.

	C	D
C	2,2	$-\frac{1}{2}, 2\frac{1}{2}$
D	$2\frac{1}{2}, -\frac{1}{2}$	0,0

In our simulations, we contrast scenarios with 2-player interactions and N-player interactions. In the N-player case, an agent participates in the dilemma with all of its immediate neighbours; the number of such neighbours determines the number of participants. In the 2-player case, an agent participates in individual 2-player games with each of its immediate neighbours.

4 Simulation Results

4.1 Experimental Setup

In these experiments, we run two sets of similar simulations, one with *2-Player* interactions the other with *N-Player* interactions, comparing the respective outcomes. It is generally accepted that when comparing the two interaction models inducing cooperation in the N-Player games is considerably harder.

The population of $A = 100$ agents is placed randomly on the $L \times L$ torus with $L = 30$, the strategies (whether to cooperate or to defect) are assigned in equal proportion, and the movement strategies are assigned randomly. A single simulation lasts 1,250 time-steps, in which the agents take 25 steps in each of 50 generations. The distribution of spatial strategies, level of cooperation, time taken for the simulation to converge on cooperation (or defection), and the total number of interactions will all be recorded. Each simulation will be run over a 1000 times to generate statistically valid results.

4.2 2-Player vs. N-Player

4.2.1 Cooperative Outcomes

On average in these environmental settings, the 2-Player interaction model is more effective at inducing the spread of cooperation in a larger percentage of simulations. We see in Table 2 that in roughly one third of evolutionary simulations using 2-player interaction, cooperation emerges as the outcome, whereas when agents participate in an N-player interaction, cooperation emerges in roughly one quarter of the simulations. On average, simulations using the 2-Player interaction model tend to converge more quickly, and with less variance. The simulations resulting in the emergence of defectors exhibit a faster convergence and less variability in convergence speed regardless of the interaction model.

Table 2: 2-Player vs. N-Player : % Cooperator Wins

	Avg	Std Dev
2-Player	33.2%	4.2%
N-Player	25.8%	4.7%

These results are in keeping with the general consensus that evolving cooperation in the N-player prisoner’s dilemma can be more difficult. This previous research did not allow movement of agents, but still captured the difficulty with

Table 3: On seeing Cooperator : % Genes Evolved

	2-Player	N-Player
Random	0%	0%
Follow	15%	27%
Flee	0%	0%
Stay	85%	73%

Table 4: On seeing Defector : % Genes Evolved

	2-Player	N-Player
Random	34%	22%
Follow	16%	2%
Flee	41%	75%
Stay	9%	1%

N-player dilemmas where an agent can exploit multiple participants and achieve a considerable gain in payoff per interaction.

4.2.2 Evolved Movement Strategies

Tables 3, 4 and 5 show the movement behaviours that are evolved for 2-player and N-player situations respectively in those runs when cooperation emerges. One hundred simulation runs resulting in cooperative outcomes are considered.

Upon seeing a cooperator in their neighbourhood, agents evolve to either stay where they are or to follow the cooperator; this occurs in both 2-player and N-player scenarios. When a defector is encountered, agents have evolved to flee or adopt a random movement in 75% of cases in the 2-player game and 97% of cases in the N-player game. For the scenarios where agents see both cooperators and defectors we see similar behaviours being evolved. Movement behaviours that promote cooperation and avoid exploitation are selected. We can see that cooperators who interact using the N-Player interaction model have a greater evolutionary incentive to be adverse to defectors.

In all cases agents learn movement behaviours that allow them to continue cooperative interactions and, to a lesser extent, to avoid interactions with defectors. Behaviours that continue defector interactions die off, although at a slower rate. Following cooperators is selected more quickly than fleeing from

Table 5: On seeing Cooperator & Defector : % Genes Evolved

	2-Player	N-Player
FollowCFollowD	27%	3%
FollowCFleeD	44%	52%
FleeCFollowD	9%	11%
FleeCFleeD	20%	34%

defectors.

It is important to note that the selective pressure to avoid defectors is removed when the defectors are replaced in the population with cooperators and hence we do not see convergence to 100% for the genes that promote avoiding defector interactions. Adopting a random movement can also often have the same effect as fleeing from or indeed following an individual.

The population did not always evolve a single strategy; random fluctuation and lack of relevant stimuli resulted in simulations in which agents converged on several strategies that were genotypically different, but phenotypically similar.

In non-cooperative runs, defectors learned to (1) follow cooperators, (2) flee from defectors, and to (3) follow both cooperators and defectors.

4.3 Variation in Density

In the previous experiments, the percentage of cooperative outcomes and the evolution of movement strategies was a function of the agent interactions. The ratio of cooperative interactions to other types of interactions influences the evolutionary trajectories.

In this experiment we aim to investigate the impact of the density of agents in the environment. We define the density as $D = A/L^2$ where A is the size of the population, and L is the length of the lattice grid. Density is a function of the population size and the size of the grid. We keep the population size constant and vary the size of the grid as a means to vary the density.

The movement strategies of agents are randomly initialized, the strategies for game interactions are assigned in equal proportions and both the movement and interaction strategies are subject to evolution. In one set of simulations, the population interacts using the 2-player interaction model, and the other uses the N-player model.

As shown in Figure 1, at the highest density level, there is not enough space within the grid for agents to move freely and so defection dominates in the vast majority of simulations. These conditions echo the traditional spatial models with an agent located in every cell where no movement is possible. These findings mirror those results with defection spreading and dominating the population.

As the density is reduced, we see that the evolutionary runs using the 2-Player interaction model are more readily able to induce higher levels of cooperation. However, using the 2-Player interaction model, random initialization in low densities can only achieve cooperation in just above 50% of simulations. With these same settings the N-Player interaction model can induce cooperation in a far greater percentage (80%) of runs.

For a grid size of 32x32 (1024 cells), the N-player interactions overtake the 2-player interaction model in their ability to induce cooperation. This result demonstrates that despite the difficulty of inducing cooperation, cooperation emerges in N-player games, the addition of movement capabilities can support the emergence of cooperation in these conditions.

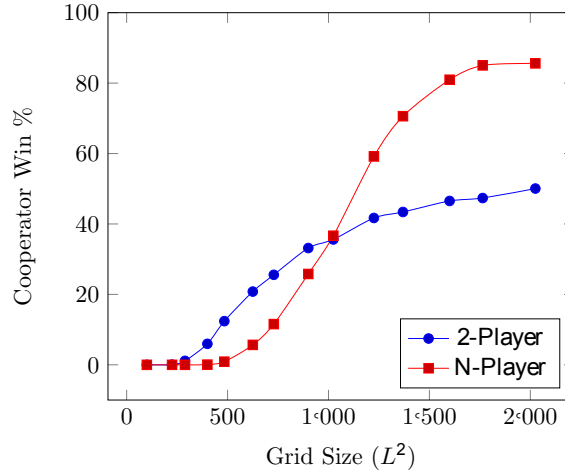


Figure 1: 2-Player vs. N-Player: The percentage of simulations resulting in cooperative victories as we vary the grid density starting from random initialization.

4.4 Seeding the Evolved Strategies

In our final experiment, the evolved movement strategies for both cooperators and defectors are seeded in the population and we repeat the density experiment. In the previous experiment both movement strategies were randomly assigned and it took several generations for movement strategies to emerge. A number of these strategies, were identified as being favorable to the emergence of cooperation. The aim of this experiment is to explore the effect of these *good* strategies when they are present in the first generation. If these strategies help cooperators to follow each other and form cooperative clusters, then higher levels of cooperation are expected across the various density levels.

Results show in both sets of simulations that the evolved cooperator movement strategies are able to induce cooperation for a much wider range of densities, as illustrated in Figure 2. There is a far greater level of cooperation than that which was achieved by either interaction model in the experiment with random initialization. For the N-player interaction model, once the grid size reaches 1024 (density roughly equal to 10%), cooperation is achieved 100% of the time. For the 2-player interaction model, this level of cooperation is also maintained for higher density levels. The agents using the N-player model are more hindered by the exploitative nature of defectors, who are also using a previously evolved movement strategy.

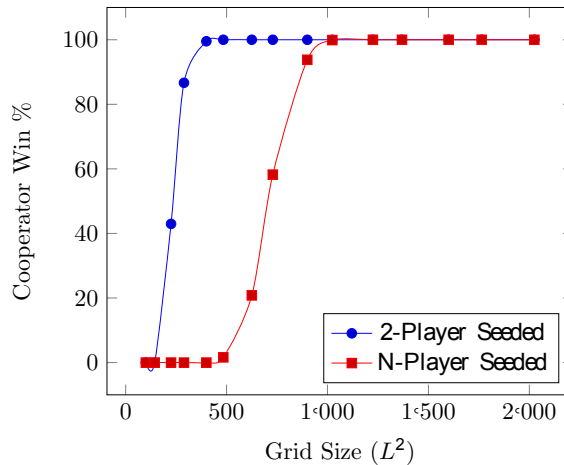


Figure 2: 2-Player vs. N-Player: The percentage of cooperative victories, as we vary the grid density, seeding the most prevalent evolved strategies for cooperators and defectors.

5 Discussion

Traditionally, it has been difficult to induce cooperation using the N-player Prisoner’s Dilemma. However, in our model we observe high levels of cooperation in a range of settings. The incorporation of a contingent mobility allows cooperators to cluster together, and avoid repeated defector interactions. In forming these clusters, these agents can increase their number of mutually cooperative interactions, thereby boosting their score. However, these cooperative clusters can be exploited by defectors unless they employ strategies that can avoid repeated exploitative encounters. We observe high levels of cooperation coupled with evolved movement strategies that encourage the formation of these larger self-preserving clusters free from the influence of defectors.

As expected, the 2-Player interaction model was more successful at inducing cooperation in the higher grid densities when we evolved from random strategies. This is due to the fact that while the chances of encountering a defector are higher, they have less of an exploitative impact on individuals or clusters of cooperators. Surprisingly, the N-player interaction model was significantly more successful at inducing cooperation when the grid density was very low. We attribute this success to the reduced chances of encountering a defector, and increased gains made by mutually cooperative interactions in clusters. Additionally, single defectors benefit by being in the neighbourhood of cooperators but this benefit is reduced in the presence of other defectors.

6 Conclusion

In this work, we introduced a model wherein agents inhabit a toroidal world, interact in social dilemmas, and have the ability to move. We explored a number of scenarios involving both the 2-player and N-player Prisoner's Dilemma. The density of agents in the grid was varied in a systematic manner, and its effect on the emergence of cooperation was discussed. We present results for 2-player and N-player interaction models and discuss the outcomes both in terms of cooperation levels obtained and in terms of the movement strategies selected.

Through experimentation, we show that the presence of contingent mobility strategies helps induce cooperation in environments where the interaction model uses the N-player Prisoner's Dilemma. We show that this cooperation can emerge for a range of density levels. Furthermore, we show that seeding a population with the previously evolved movement strategies results in very high levels of cooperation in both the 2-player and the N-player interaction models for a large range of densities. These simple mobile strategies are extremely adept at spreading cooperation throughout a mobile population without the need for complex computation, or costly memories.

Future work will involve a more thorough investigation of the nature of the cooperative clusters that form throughout the evolutionary runs. We also wish to explore a larger set of N-player social dilemmas and explore a more expressive spatial topologies.

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