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Pile group settlement estimation: the suitability of nonlinear interaction factors

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Abstract

In this paper, predictions of pile settlement determined by appropriate superposition of two-pile interaction factors are compared with those computed from continuum analysis. PLAXIS 3D Foundation is used in conjunction with the non-linear Hardening Soil model and pile group sizes studied are greater than in previous research. The study is presented in two phases: (i) examination of the modulus variation within the group allowed the elimination of some variables and (ii) a comparison of pile settlement predictions in the context of the significant variables. The research shows that predictions using the former approach match the continuum analyses very closely for friction piles and reasonably well for end-bearing piles and show potential for reducing computational time and effort in such problems. The paper also distinguishes between alternative definitions of interaction factor and shows that most accurate predictions arise from interaction factors calculated when only one of the two piles is loaded.

Keywords: pile groups, settlement, nonlinear, superposition, interaction factor

Introduction

Various methods have been used to predict pile group behavior, including the boundary element method (e.g. Basile 2003, Suleiman and White 2006), the finite element (FE) method (e.g. Comodromos and Bareka 2009) and more recently, methods based on mobilizable strength design principles (e.g. Shen and Teh 2002, Klar and Leung 2009). While these continuum methods consider the unified response of the pile-soil system, they can become computationally intensive if soil stiffness nonlinearity, a large number of piles and/or non-uniform geometries are also to be modeled. The ‘interaction factor method’ (IFM), attributed to Poulos (1968), remains one of the simplest means of estimating load interaction among group piles. This method involves the consideration of a pair of piles (a ‘load source’ pile and a ‘load receiver’ pile) initially; the interactive displacements experienced by the receiver pile originating from the loaded source pile are computed. If this exercise is performed for the various pile spacings relevant to a group configuration, superposition may then be used to calculate the total interactive displacements induced by each pile in the group on all other piles. Two alternatives for calculating the two-pile interactive displacements feature in the literature; the receiver pile may be load-free (henceforth referred to as Approach I; see Fig. 1a) or loaded (henceforth referred to as Approach II, see Fig. 1b).

The literature documents many examples, based on linear elastic (LE) analyses, of the successful application of IFM to groups with relatively small pile numbers. Lee (1993) utilized Approach II IFM to extrapolate the results of a simplified single pile analysis to pile groups of up to 16 piles and results compared well to boundary element predictions (Butterfield and Banerjee 1971). Similarly, Poulos (1988) used modified Approach II interaction factors to predict the responses of the groups of 4, 5 and 9 piles documented by O’Neill *et al.* (1981) with satisfactory results. The modified interaction factors accounted for a reduced ‘near-pile’ soil modulus in an attempt to simulate the degradation of soil modulus with stress in the vicinity of a loaded pile.

Although the principle of superposition is not strictly valid in nonlinear engineering problems, Leung *et al.* (2010) noted that while the load-displacement response of an individual pile is essentially nonlinear, the interaction effects between piles remain largely elastic. Jardine and Potts (1988) applied interaction factors derived from Approach I-type nonlinear analyses documented by Jardine *et al.* (1986) to a 8-pile group with satisfactory agreement to measured field data. These results were further verified by Ganendra (1994) using a nonlinear 3-D analysis in the ‘FSAM’ program. Furthermore, Zhang Q-Q *et al.* (2010) applied Approach I IFM to the 9-pile group load test documented by Koizumi and Ito (1967), also with satisfactory agreement. However, it is clear that much of the justification for

the applicability of nonlinear IFM has come from very small pile groups and the influence of various pile/soil parameters on the successful application of nonlinear IFM is unknown.

In addition, some authors suggest that the use of interaction factors based solely on the spacing of any pair of piles in a group may overlook the potential reinforcing effects of intervening piles (Basile 2003; El Sharnouby and Novak 1990; Mylonakis and Gazetas 1998; Southcott and Small 1996). El Sharnouby and Novak (1990) compared predictions of stiffness efficiency (i.e. the ratio of the settlement of a single pile to that of a group pile at the same load per pile) of rigidly-capped pile groups determined by using (Poulos and Davis 1980) interaction factors with those from a continuum analysis based on the stiffness method and Mindlin's equation (Mindlin 1936). The authors report that predictions agree well for floating pile groups but diverge as the soil stiffness beneath the pile base increases, particularly for larger group sizes. They attribute this to the failure of the IFM to account for the reinforcing effects of intervening piles; although the study was limited to small pile groups considered within a LE framework.

The purpose of this paper is to explore, using FE analysis, the suitability of the IFM for the estimation of the nonlinear settlement of pile groups of significant size for various pile/soil parameters. PLAXIS 3-D Foundation is used in conjunction with the Hardening Soil (HS) model for this purpose. The primary focus is on the accuracy of IFM using Approach I since this is the more widely-accepted approach. However, Approach II has never been considered within a nonlinear framework, to the knowledge of the authors, so this paper affords the opportunity to compare both approaches. The analysis presented has two components:

- (i) A parametric study aimed at reducing the number of variables considered in the subsequent settlement analyses in Sections 6 and 7, thereby reducing the computational time associated with the study. This involves a comparison of the soil modulus regimes predicted between (a) a pile pair and (b) piles along a cross-section within a group. The modulus regimes provide a measure of the stress fields between piles. The following potential influences upon this comparison were considered: pile cap fixity conditions (rigidly-capped and non-capped groups are examined as extremes to the full spectrum of cap flexibility), soil type and the pile length-to-diameter (L/D) ratio; these are subsequently shown to be relatively insignificant and are therefore excluded from settlement analyses.
- (ii) Group settlements determined by (a) two-pile interaction factors with appropriate superposition and (b) direct analyses of complete groups are compared. Variables included pile spacing, load factor, number of piles and pile base stiffness.

The default HS soil parameters used pertain to the soft clay/silt at the well-publicized geotechnical test bed at Belfast, Northern Ireland. However, in the course of the parametric study, variations in some of the parameters have meant that the scope of the study is broader than merely soft clays. In addition, a limited number of analyses are carried out based on parameters for stiff Boston Blue Clay as an alternative clay type.

Details of the finite element modeling

Details of FE parametric study

The pile/soil parameters considered in the study are illustrated in Fig. 2. The default pile length (L) and diameter (D) are 6.0 m and 0.282m, respectively (the diameter gives an equivalent pile area to that of a square pile of width 0.25 m), corresponding to those tested by McCabe and Lehane (2006). E_1 is the stiffness of the upper layer, E_2 is the stiffness of the lower layer, and the boundary between them is located at a depth h below ground level ($h \geq L$). In subsequent analyses, a value of $h/L = 3$ was maintained except in section 7.2 where $h/L = 1$. The depth below ground level to the bottom mesh boundary, H , was chosen as $3L$ so that the lower mesh extremity had no effect on the FE output. Likewise, the lateral (roller) boundaries of the FE model for each analysis were located at a distance $25D$ from the outer row of piles such that no influence was recorded on output.

Other features of the model are shown in Fig. 3 (the illustration is for a non-capped 16-pile group). 15-node wedge elements were used in the study comprising 6-node triangular elements in the horizontal direction and 8-node quadrilateral elements in the vertical direction. Symmetry was exploited to reduce the number of elements used in the mesh and associated computational time which varied with group size with a maximum of ~90,000 elements for a 196-pile group (the limiting size for this study). In all analyses, the mesh was refined in zones of high stresses near the piles. Coarse, medium and fine meshes were used to confirm mesh convergence for all analyses.

Interaction factors

The interaction factor (α) is defined by Poulos (1968) as:

$$\alpha_{ij} = \frac{s_{ij}}{s_{ii}} \quad (1)$$

where s_{ii} is the settlement of pile i when loaded alone (by load P_i), i.e. a single pile, and s_{ij} is the additional settlement of pile i due to load P_j acting on nearby pile j .

The value of s_{ij} for a pile pair in Eq. (1) can be calculated using Approaches I and II as detailed below.

Approach I – s_{ij} is determined, assuming pile i is not loaded (Fig. 1a), for each value of s/D and the corresponding value of α_{ij} is calculated. In this case the relative shear stress at the pile-soil interface of pile i is relatively small and therefore there is no soil modulus degradation at pile i .

Approach II - s_{ij} is determined, assuming pile i is loaded (Fig. 1b), for each value of s/D and the corresponding value of α_{ij} is calculated. Due to the large shear strains at the pile-soil interface for pile i , soil modulus degradation occurs in the vicinity of piles i and j as shown in Fig. 1b.

The 2-pile interaction factors used in the present study were obtained from PLAXIS FE output according to the following procedure; stages (i)-(iv) are common to Approaches I and II:

- (i) Interface elements are assigned in the soil model to allow for pile-soil slip.
- (ii) Initial stresses are generated by the K_0 procedure, a special calculation method available in PLAXIS.
- (iii) The concrete pile installation was reflected by changing appropriate elements to a linear elastic material with a Young's modulus of 30 GPa and a Poisson's ratio, ν , of 0.15.
- (iv) Pile i was loaded by placing a compressive uniform distributed load along the top surface of the pile material and recording s_{ii} . Since it is only the settlement of a single pile under its own load that is required, there is no pile j in this scenario.
- (v) Both piles i and j are now considered. In the case of Approach I, pile j was loaded and s_{ij} recorded. In the case of Approach II, piles i and j were loaded and s_{ij} recorded. P_i and P_j will always be equal.

Superposition of 2-pile interaction factors to predict full pile group behavior

The calculation of the settlement, s_i , of a pile i within a group of N piles by the IFM may be obtained by using the formula below (Poulos 2006):

$$s_i = \sum_{j=1}^{j=N} (P_j s_1 \alpha_{ij}) \quad (2)$$

where P_j = load on pile j ; s_1 = settlement of single pile per unit load ($=s_{ii}/P_i$); α_{ij} = interaction factor for pile i due to a loaded pile j within the group, corresponding to the centre-to-centre spacing between piles i and j ; N = number of piles in the group. The values of α_{ij} used in Eq. (2) above are obtained by incorporating PLAXIS output into Eq. (1). In the settlement analyses presented later in this paper

(Section 6), s_{IFM} is used to represent the value of s_i for a central pile (the centre pile if N is odd, one of four equal central piles if N is even).

Calculation of s_i for non-capped pile groups is relatively straightforward because the pile head load is known; individual s_{ij} contributions are merely summed. Calculation of the settlement of a rigidly-capped group (i.e. for the case of equal settlement of all piles) is considerably more involved; the pile displacements are equated forming N simultaneous equations and can thus be solved for the unknown loads P_j from which the overall settlement of the group may be derived. A simple algorithm was developed in *Mathematica* for this purpose, the code for which is provided in Appendix A.

'Direct' analyses

The term 'direct analysis' is used to describe those in which pile groups were modelled as a continuum. Rigid pile groups and non-capped (representing infinitely flexible) pile groups were considered.

For rigid pile groups the following analysis procedure was followed:

- (i)-(iii) These steps are the same as the corresponding steps in the Section 2.2.
- (iv) Excavation of soil to a depth of 0.5 m below the pile heads. For a free-standing pile group in PLAXIS, it is necessary to excavate the soil below the pile cap so that it does not come into contact with the ground surface. To ensure that the excavation of the soil in stage (iv) did not induce changes to the initial stresses in the soil, a dummy material was employed to a height of 0.5m above the soil profile with weight density $\gamma=0$ kN/m³ as shown in Fig. 2.
- (v) Installation of the pile cap (modelled as a 'floor' in PLAXIS) along the top of the pile group. Floors in PLAXIS are composed of 6-noded triangular plate elements. The same properties employed for the concrete pile in (iii) were used for the cap.
- (vi) Pile group loading by placing a compressive uniform distributed load along the top surface of the pile cap.
- (vii) Recording of the pile cap displacement s_D (s_D denotes settlement determined by a direct analysis) versus pile head load (zeroed at the start of loading); s_D corresponds to the overall group settlement for rigidly-capped pile groups and the settlement of the central pile for non-capped groups.

The term rigid pile group is used somewhat loosely to denote a pile group connected to a stiff pile cap since it is acknowledged that in general, the pile cap may undergo slight deformation for larger pile group sizes. However, substantial cap rigidity was confirmed by checking the differential settlement between corner and centre group piles.

The stages used in the analyses of flexible pile groups are similar to the stages for 2-pile interaction factors described in section 2.2.

Applicability to piled rafts

In practice, piled rafts are commonly used in conjunction with larger pile groups. The IFM is strictly not applicable to situations where the pile cap is in contact with the ground. However, given that the majority of piled rafts are designed ignoring any contribution from raft-soil interaction (Horikoshi and Randolph 1998), the analyses presented herein will be of interest to geotechnical designers. Moreover, a reliance on the contribution of the raft-soil resistance can lead to unconservative design in situations where the occurrence of ground surface settlement and/or scour by water current, for example, is possible.

Constitutive model and parameters

The Hardening Soil (HS) model

The nonlinear Hardening Soil (HS) model, adopted to model both the behavior of the fill and *sleech*, has the advantage (over an elastic perfectly-plastic model) that the yield surface is not fixed in principal state but instead can expand due to plastic straining (Brinkgreve 2007). The HS model is an improvement on the Duncan-Chang hyperbolic model in that the theory of plasticity is used as opposed to the theory of elasticity. It also incorporates soil dilatancy and a yield cap although creep behavior is not considered. Further details are available elsewhere e.g. Schanz *et al.* (1999), Sheil and McCabe (2013).

Belfast soil parameters

The majority of analyses reported in this paper are based upon the stratigraphy at a heavily-researched soft clay/silt geotechnical test bed in Belfast, Northern Ireland. The stratigraphy consists of a layer of made ground which extends to a depth of ~ 1.0m, a layer of silty sand from 1.0m to 1.7m, and a lightly overconsolidated soft estuarine silt (locally known as *sleech*) to a depth of 8.5 m. A stratum of medium dense sand exists at 8.5 m below ground level and the water table was found at approx. 1.4 m below

ground level (with a small tidal variation). The summary properties of the *sleech* given in Table 1 are based on laboratory tests quoted in Lehane (2003) and McCabe and Lehane (2006). The interpreted HS parameters have been provided in Table 2. Sheil and McCabe (2013) have successfully predicted the behavior of a floating single pile and 5-pile group at the site (documented by McCabe and Lehane 2006) using these parameters. Variations upon the values of E_2 (defined in Fig. 2) shown in Table 2, required in simulations where $E_2/E_1 > 1$ (section 7.2), are provided in Table 3. Further details on the development and validation of the HS model parameters are provided in Sheil and McCabe (2013).

Boston Blue Clay (BBC) soil parameters

While the majority of the parametric analyses in the subsequent sections are based on the aforementioned Belfast soil profile, an alternative soil type (Boston Blue Clay, BBC) was also considered (in section 5.5 only) to assess the wider applicability of the findings of the present study. Reference was made to Altabaa and Whittle (2001), Konstantakos *et al.* (2005) and Ladd *et al.* (1999) for the selection of the HS parameters listed in Table 2.

Influence of soil nonlinearity on pile-soil-pile interaction

Elastic interactive displacements

PLAXIS load-displacement responses for a pile pair, using both LE and HS models, are compared in Fig. 4 for a single loaded pile j and a ‘load free’ pile i at $s=3D$, i.e. using Approach I (Fig. 1a). The curves annotated ‘ $P_{j-s_{jj}}$ ’ relate load to displacement on pile j ; whereas those annotated ‘ $P_{j-s_{ij}}$ ’ relate the load on pile j to the displacement of pile i . In the LE analysis, the Young’s modulus of the soil was chosen based on the initial stiffness of the nonlinear load-displacement response of the loaded pile. Since the HS model predicts linearly increasing soil stiffness with depth (i.e. a Gibson soil), a similar profile with depth was adopted in the LE soil model.

It can be seen from Fig. 4 that pile displacement determined by the HS model and the LE soil model are almost identical for pile i which further supports the theory that pile-to-pile interaction is essentially a linear phenomenon (Chow 1986; Leung *et al.* 2010; Mandolini *et al.* 2005; Randolph 1994). This theory is used as a basis to assess the applicability of the IFM to nonlinear analyses in the proceeding sections.

Differences in 2-pile interaction factors from Approaches I and II

For LE analyses, predictions of α_{ij} using Approaches I and II will yield the same result. In the case of nonlinear analyses, potential differences between Approaches I and II have not been explored in the literature. These differences are examined here, using the HS constitutive model. In Fig 5, predicted values of α_{ij} using both Approach I and Approach II are plotted against s/D or s/D_{eq} , with the results of alternative prediction methods and field data included for comparison. A value of LF (defined as applied load as a proportion of single pile capacity) equal to 0.4 has been adopted in Fig. 5; the pile capacity was defined nominally at a pile head displacement of $0.1D$.

It can be seen from Fig. 5 that the new PLAXIS analyses and the curve from the 2-D nonlinear FE analyses documented by Jardine *et al.* (1986) show much improved agreement to field data than the predictions determined using the *PIGLET* computer program (Randolph 2003) and the approach documented by Chen *et al.* (2011) where the soil is idealised as a LE medium. The method employed by Chen *et al.* (2011) differs from conventional approaches in that a more rigorous approach to consider pile-soil interaction is proposed using the fictitious pile-extended half-space model. Although comparisons with the data reported by Cooke (1974) and Caputo and Viggiani (1984) are only indicative (since different soil and pile properties as well as load levels will lead to differences in interaction factors), PLAXIS results show good agreement to the measured field data beyond a value of $s/D_{eq}=2.5$. The interaction factors presented in Fig. 5 form the basis for corresponding settlement predictions later in the paper.

In Fig. 6, the percentage difference in two-pile interaction factor predictions ($\Delta\alpha$) between Approaches I and II has been plotted against LF values ranging between 0 and 0.67 and for a range of s/D values. In this paper, $\Delta\alpha$ is defined as:

$$\Delta\alpha = \left(\frac{\alpha_{II} - \alpha_I}{\alpha_{II}} \right) \times 100 \quad (3)$$

where α_I and α_{II} are the interaction factors calculated using Approaches I and II, respectively (the ij subscript has been dropped for clarity). From Fig. 6, it can be seen that, although the value of $\Delta\alpha$ is relatively insignificant for a LF of 0.25, an increase in the load level to the higher LF values of 0.4 and 0.67 results in a notable difference in interaction factor predictions. This divergence in α_I and α_{II} predictions may lead to a significant difference in settlement estimation when combined with the IFM for larger group sizes.

Evaluation of IFM approaches – modulus study

Overview

The primary goal of this paper is to compare s_{IFM} (i.e. s_i in eqn [2], for a central group pile) and corresponding s_D values (direct analysis) derived from FE modelling. However, a comprehensive comparison which considers appropriate ranges of all other important pile group variables would be prohibitive from a computational viewpoint. For example, the computational time required to gather all data for the $N=196$ group represented by a single datapoint in Figs. 13-15 is approximately 40 hours for the 1.6 GHz quad core i7 processor used. In this preliminary study, variables are considered which were ultimately eliminated from the parametric study presented in Sections 6 and 7. These variables include pile cap fixity conditions, soil type and the pile length-to-diameter (L/D).

In this preliminary parametric study, the variation of Young's modulus between piles is used as a measure of the intervening stress fields. E is used to denote the modulus of the soil at a particular radial distance r from the center pile of the group while E_0 is used to denote the far-field soil modulus. Representative profiles of the following variations of E/E_0 with radius are considered:

(i) between a pile pair and

(ii) between piles along a cross-section within a conventional $\sqrt{N} \times \sqrt{N}$ square pile group (see Fig. 7). Results for a $N=25$ group are presented here. This group size was chosen for illustrative purposes; however similar trends were observed for alternative group sizes.

In these analyses, an average soil modulus between ground level and $L=6.0$ m was considered since the current pile/soil parameters are representative of friction piles; the average soil modulus is compared to that at different depths in Fig 8. In this context, it is the agreement between the IFM and direct predictions at the interface of the soil-receiver pile that is of interest, since it is well established that soil properties at the pile-soil interface govern pile behavior e.g. Lee *et al.* (2002).

Influence of pile cap conditions

The distribution of E/E_0 has been plotted against r/R in Fig. 9 for groups with a rigid cap and no cap for a typical value of $LF=0.4$ and for a spacing between group piles of $(s/D)_g=3$. In addition, the variation in E/E_0 predicted between just two piles (these are highlighted by shading) using Approaches I and II ($(s/D)_{IFM}=3$) have also been superimposed on Fig 9. It can be seen that the conditions imposed at the pile head have little influence on the soil modulus regime existing within the group determined by the direct analyses. In light of this, the authors adopt the non-capped groups as the basis for the

parametric study in the following sections for computational savings. It is also significant that the predictions of E/E_0 at the soil-receiver pile interface match very well with those of Approach I, while Approach II produces lower E/E_0 values. It can also be seen that the soil modulus between piles in the direct analyses exceeds the far-field value; this can be attributed to the increased mean effective stress (p') between piles, similar to that documented by Reul (2004) for a free-standing pile group. As mentioned, however, it is the conditions at the pile interface that are of interest in this study.

Influence of pile length

The dimensionless expression for the pile slenderness ratio documented by Mylonakis and Gazetas (1998) considers both the relative pile-soil stiffness and pile length. However, such an expression is not convenient to use where the soil stiffness varies with stress level. Therefore, in this study the authors have adopted the pile length-to-diameter (L/D) ratio as a more expedient means of considering pile slenderness. The influence of L/D can be observed by comparing Figs. 9 and 10; in the latter, L/D has been increased to 50 by reducing the default pile diameter. It can be seen that the distribution of E/E_0 is largely unchanged while Approach I remains the preferred IFM approach.

Influence of δ/φ' (R_{inter})

The value of R_{inter} in the clay layer has been maintained at 0.55 in Section 5. Although these values are common for pile-clay interfaces, the possibility of zero strength reduction at the interface, i.e. $R_{inter}=1$, has been considered in Fig. 11. It can be seen that the influence of R_{inter} on the results is negligible. The authors have chosen not to examine the influence of this parameter hereafter.

Influence of soil type

As a check that the findings of the present study are not unique to the particular soil properties adopted, a completely different soil profile has been considered on the group modulus distribution. In Fig. 12, the analyses are based upon the HS parameters of the well-documented BBC (see section 3.3). The conclusions drawn from the BBC profile are very similar to the previous results for the same geometric parameters in Fig. 9. Thus it can be deduced that the soil type considered in the analyses is of secondary importance when compared to group geometry and is therefore not considered in sections 6 and 7.

Rigidly-capped group validation

In this section, PLAXIS 3-D is used to determine the suitability of IFM in conjunction with superposition for predicting the settlement of rigidly-capped (square) pile groups. A uniformly-distributed load (representing an average LF per pile of 0.4) was applied to the surface of the pile cap in the direct analysis. The variation in s_{IFM}/s_D (defined in sections 2.3 and 2.4, respectively) with the number of piles in the group has been plotted in Fig. 13 where a value of $s_{IFM}/s_D = 1$ indicates perfect agreement between results. Subscripts I and II are used for s_{IFM} to differentiate between settlements predicted using Approaches I and II.

It can be seen that IFM settlement predictions determined using Approach I ($s_{IFM(I)}$) agree well with direct predictions (s_D) where the entire group is modeled, with a maximum difference of ~6%. In contrast, predictions determined using Approach II ($s_{IFM(II)}$) fall significantly below s_D and appear to diverge with N to ~11%. For the sake of computational efficiency, the validation of IFM while considering the influence of various pile/soil parameters is hereafter restricted to non-capped groups.

Non-capped group validation

Superposition of nonlinear interaction factors ($h/L=3$)

PLAXIS 3-D is used to determine the ability of the IFM in conjunction with superposition to predict the settlement of non-capped pile groups. A load of 25 kN (representing a LF of 0.4) was applied to the head of each pile in the direct analysis. The variation in s_{IFM}/s_D with N (up to $N=196$) has again been plotted in Fig 14 for s/D values of 2 (Fig 14a), 3 (Fig 14b) and 5 (Fig 14c).

In all cases, $s_{IFM(I)}/s_D$ values lie between 0.9 and 1.1, while $s_{IFM(II)}/s_D$ values fall significantly below 1 and reduce with N . In Fig. 14d, the suitability of Approach I is further evaluated further by varying LF ($s/D=3$). Although $s_{IFM(I)}/s_D$ deviates most from unity at LF=0.67, the deviation does not exceed 10%. In any event, a value of LF=0.4 is deemed to be more suitable serviceability criterion for piles in practice.

Group 'reinforcing' effects ($h/L=1$)

El Sharnouby and Novak (1990) compared predictions of the settlement performance of rigidly-capped pile groups determined by the IFM (Poulos and Davis 1980) with those from a continuum analysis based on the stiffness method and Mindlin's equation (Mindlin 1936). The authors report that predictions agree well for floating pile groups but for a value of $E_2/E_1=10$, those authors documented

an over-prediction of rigid pile group settlement by the IFM of 28%, 49% and 64% for a $N=9$, $N=25$ and $N=100$ pile groups respectively, which they attributed to the reinforcing effects of intervening piles.

Therefore for group sizes up to $N=196$, s_{IFM}/s_D was determined for values of E_2/E_1 ranging from 1 to 50. For the purpose of these analyses, a stiff bearing stratum has been accommodated in the soil model at $h/L=1$. The soil properties of the stiff bearing stratum are otherwise similar to those adopted for the soft clay except that the soil stiffnesses have been multiplied by a factor of E_2/E_1 and are presented in Table 3.

It can be seen from Figs. 15a-15c that although there appears to be a slight divergence in s_{IFM}/s_D from unity for large group sizes, the divergence is not nearly as pronounced as that reported by El Sharnouby and Novak (1990). A maximum difference of $\sim 20\%$ was observed for the combination of $N=196$ and $E_2/E_1=50$. This divergence can be attributed to the failure of two pile interaction factors (using either approach) to replicate the increase in E/E_0 (due to an increase in p') beneath the base of the pile group as shown in Fig. 16 where E/E_0 represents the average distribution over a depth of 2.0 m beneath the base of the piles. Moreover, it is now common knowledge that interaction between pile bases is significantly less than interaction between pile shafts (see for example Randolph and Wroth 1979); this leads to significantly reduced pile group settlement predicted by a direct analysis.

Conclusions

A numerical study on the applicability of nonlinear interaction factors to pile group settlement analysis has been presented using PLAXIS 3-D Foundation in conjunction with the advanced nonlinear Hardening Soil model representing ground conditions at a soft clay/silt site. The authors conclude as follows:

- i) Predictions of the soil modulus at the pile-soil interface of a designated receiver pile determined using Approaches I and II were compared to those predicted by a continuum analysis. Results show that for the range of parameters considered, Approach I showed satisfactory agreement to that predicted within groups. Approach II, however, consistently under-predicted the soil modulus at the pile-soil interface of the receiver pile.
- ii) 'Rigidly-capped' pile group settlements determined by the IFM using Approach I agree well with direct predictions for values of N ranging between 4 and 196. In contrast, predictions determined by Approach II tend to be strongly unconservative.

- iii) Flexible pile group settlements determined by the IFM using Approach I and direct analyses agree well for values of s/D ranging between 2 and 5 thus substantiating the findings of the modulus study. Although an increase in load level slightly reduces the accuracy of Approach I, predictions remain within 10% of direct group predictions.
- iv) In addition, it is shown that for a value of $E_2/E_1 = 50$ the IFM over-predicts pile settlement by a maximum of 20% compared to a direct analysis for groups of up to 196 piles, significantly less than the divergence reported by El Sharnouby and Novak (1990) for small rigid pile groups (~64% for a 100-pile group). This is attributed to the increase in E/E_0 below the pile bases which cannot be accounted for using the IFM.
- v) The present findings suggest that predictions determined by the IFM provide sufficient agreement to guide predictions of the settlement of groups of up to 200 piles when using Approach I thus implying the possibility of saving considerable time and computing requirements in practice.

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Appendix A – *Mathematica* IFM code

```
ClearAll["Global`*"] (*Clear all variables, local and
global*)
SetDirectory["C:\\file_location"]; (*file directory*)
intimport=Import["matrix_filename.csv"]; (*Import interaction matrix*)
int_matrix=intimport;
n=100; (*number of piles*)
delta=Table[d,{z,n}]; (*displacement vector*)
loads=Table[p[z],{z,n}]; (*load vector*)
flattened=Flatten[{loads,d}];
pave=Sum[p[z],{z,n}]/n;
Solve[delta==loads.int_matrix && pave==21, flattened]
```

Table 1 Typical properties of the Belfast *sleech*

Clay Fraction (%)	20 ± 10
Fines Content (%)	90 ± 5
Water Content (%)	60 ± 10
Plasticity Index (%)	35 ± 5
Organic Content (%)	11 ± 1
Peak Vane Strength (kPa)	22 ± 2
Over-Consolidation Ratio (OCR)	1.1 to 2.0
Friction Angle (°)	33 ± 1

Table 2 Belfast soil parameters used in HS model

	Fill	Silty sand	Sleech	Lower sand	BBC
Depth (m)	0.0-1.0	1.0-1.7	1.7-8.5	8.5-12.0	0-12
Sat weight density γ_{sat} (kN/m ³)		17.5	16.5	20	18.5
Unsat. Weight density γ_{unsat} (kN/m ³)	18.5	17.5	16	20	18
Horiz. Permeability k_x (m/day)	1.0	$2.6 \cdot 10^{-5}$	$5.2 \cdot 10^{-5}$	8.64	$5 \cdot 10^{-5}$
Vert. permeability k_y (m/day)	1.0	$2.6 \cdot 10^{-5}$	$2.6 \cdot 10^{-5}$	8.64	-
Friction angle, ϕ' (°)	33	33	33	40	33.4
Dilatancy angle, Ψ (°)	0	0	0	0	0
Cohesion, c' (kPa)	1.0	1.0	1.0	1.0	1.0
Pre-overburden pressure, POP (kPa)	15	-	-	-	-
Overconsolidation ratio, OCR	-	2	1.2	2	2
Coefficient of lateral earth pressure, K	0.46	0.66	0.5	0.46	0.48
Tangent oedometric stiffness, $E'_{\text{oed}}^{\text{ref}}$ (Mpa)	25	12	6.5	75	30
Secant stiffness in drained triaxial test, E'_{50}^{ref} (Mpa)	25	12	6.5	75	30
Unloading/reloading stiffness, $E'_{\text{ur}}^{\text{ref}}$ (Mpa)	75	36	19.5	225	90
Reference pressure for stiffness, p_{ref} (kPa)	30	30	30	30	100
Power for stress-level dependency of stiffness, m	0.5	0.5	1.0	0.5	1
Interface strength reduction factor, R_{inter}	0.9	0.9	0.55		0.6

* $E'_{\text{oed}}^{\text{ref}}$ set equal to E'_{50}^{ref} in the absence of an appropriate reference

Table 3 Stiff bearing stratum properties

Parameter	E ₁	E ₂			
		E ₂ /E ₁ =	10	30	50
φ' (°)	33		33	33	33
Ψ (°)	0		0	0	0
c' (kPa)	1.0		1.0	1.0	1.0
POP (kPa)	0		0	0	0
OCR	1.2		1.2	1.2	1.2
K	0.5		0.5	0.5	0.5
E' _{oed} ^{ref} (Mpa)	6.5		65	195	325
E' ₅₀ ^{ref} (Mpa)	6.5		65	195	325
¹ E' _{ur} ^{ref} (Mpa)	19.5		195	585	975
p _{ref} (kPa)	30		30	30	30
m	1		1	1	1

¹ It is assumed $E'_{ur}{}^{ref} = 3 * E'_{50}{}^{ref}$ (Brinkgreve 2007)

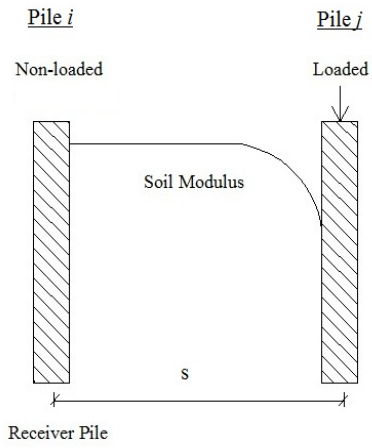


Fig. 1a Illustration of Approach I

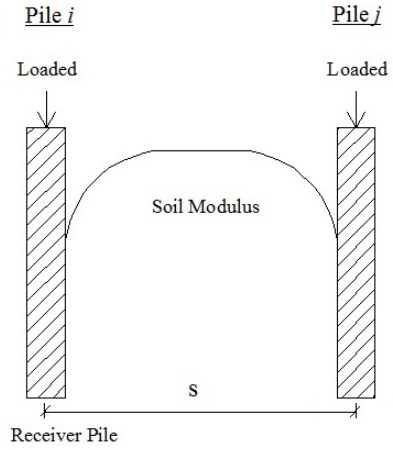


Fig. 1b Illustration of Approach II

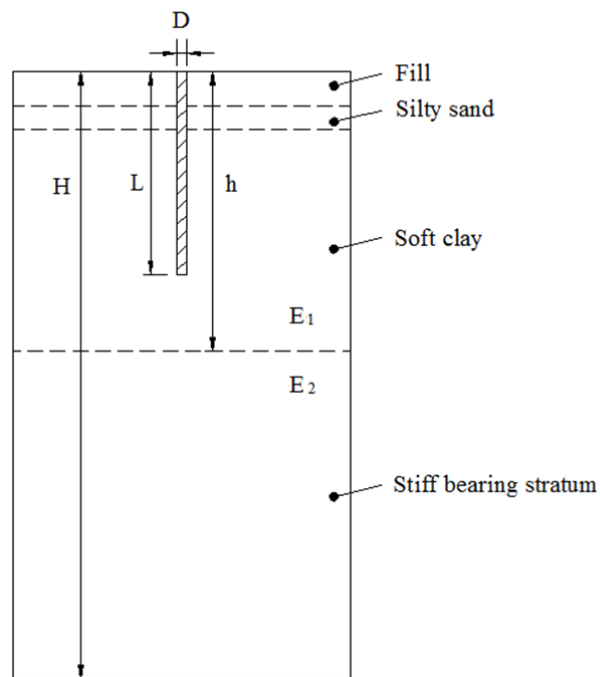


Fig. 2 Illustration of pile/soil parameters

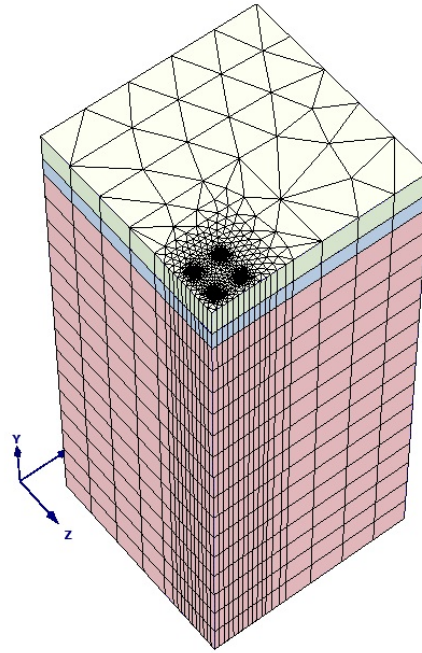


Fig. 3 Finite element mesh for 16-pile group

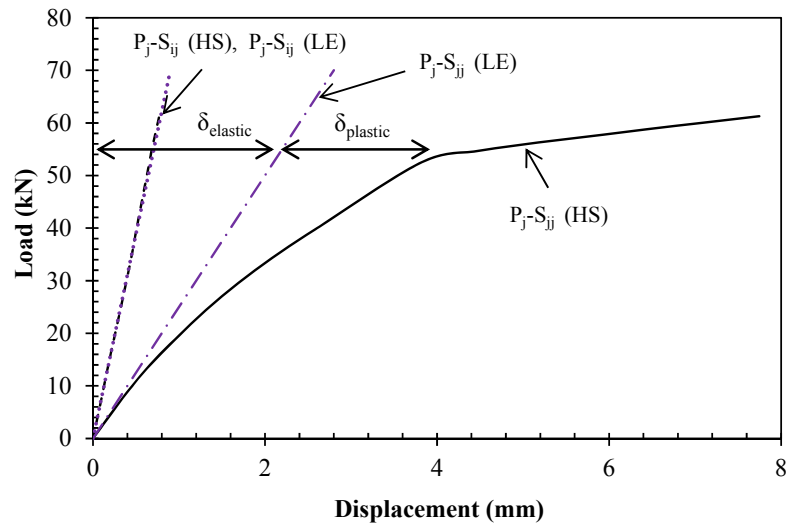


Fig. 4 Predictions of load-displacement response of loaded and 'non loaded' piles; $s/D=3$, $h/L=3$

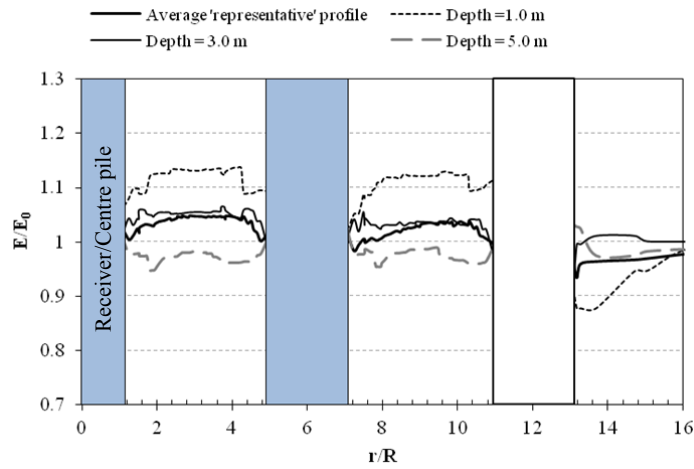


Fig. 8 Comparison of average group distribution of E/E_0 to distribution at various depths; $LF=0.4$

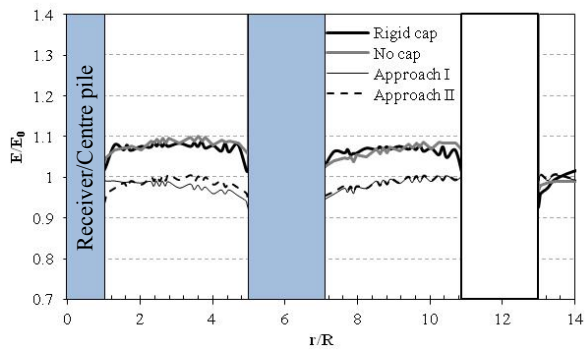


Fig. 9 Comparison of predicted variations in E/E_0 ; $LF=0.4$, $N=25$, $(s/D)_g=3$, $(s/D)_{IFM}=3$

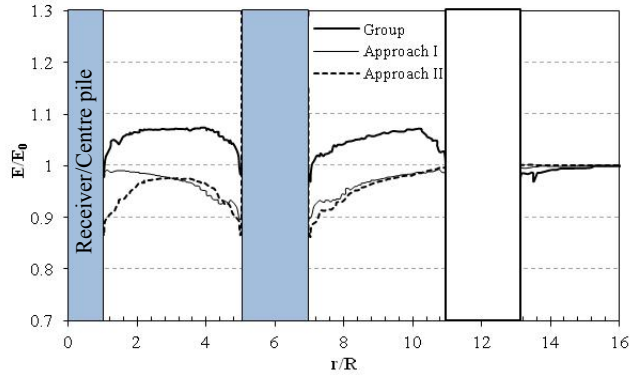


Fig. 10 Comparison of predicted variations in E/E_0 for $L/D=50$; $LF=0.4$, $N=25$, $(s/D)_g=3$, $(s/D)_{IFM}=3$

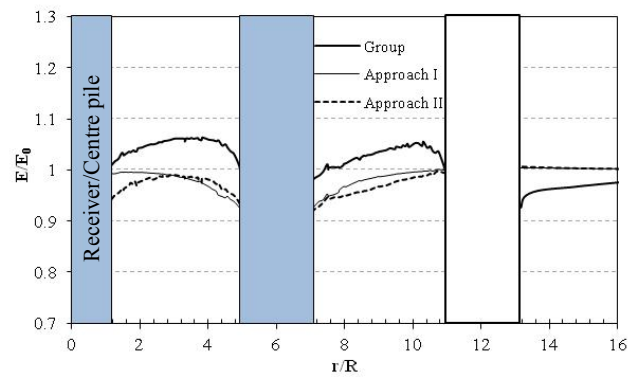


Fig. 11 Comparison of predicted variations in E/E_0 for $R_{inter}=1$; $LF=0.4$, $N=25$, $(s/D)_g=3$, $(s/D)_{IFM}=3$

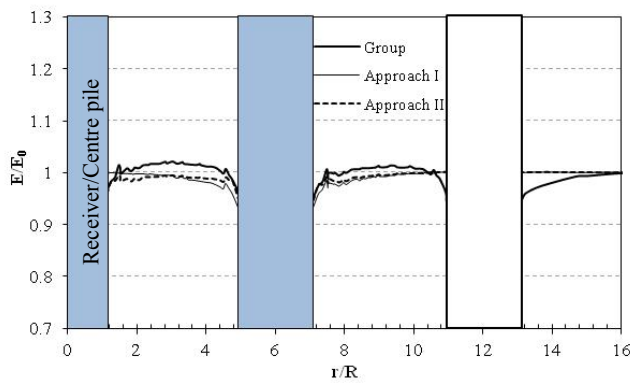


Fig. 12 Comparison of predicted variations in E/E_0 for BBC; $LF=0.4$, $N=25$, $(s/D)_g=3$, $(s/D)_{IFM}=3$

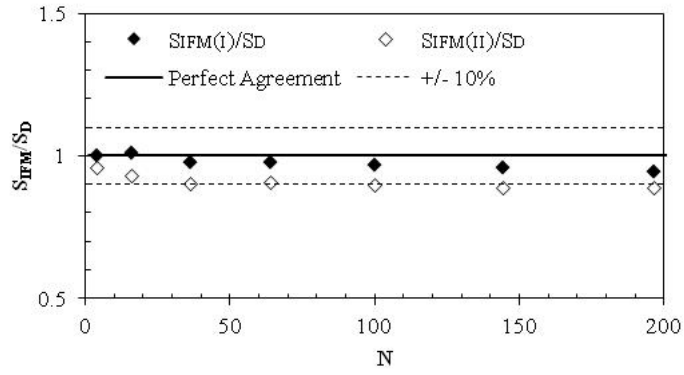


Fig. 13 Comparison between settlement predictions determined by IFM and a direct analysis for rigidly-capped groups; $h/L=3$, $s/D=3$

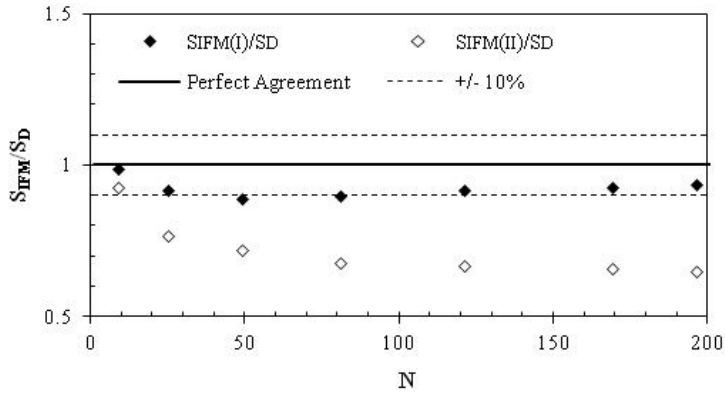


Fig. 14a Comparison between settlement predictions determined by IFM and a direct analysis for $s/D=2$

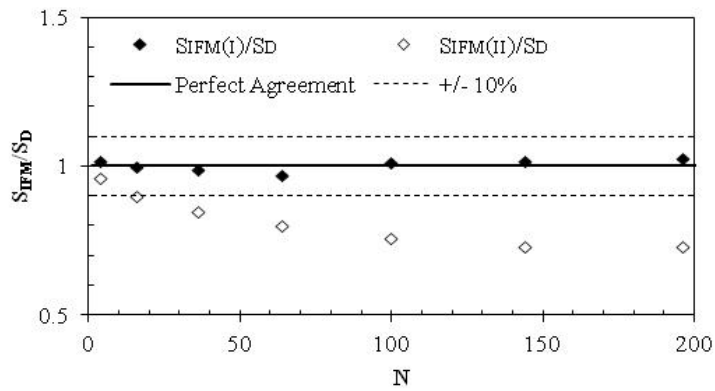


Fig. 14b Comparison between settlement predictions determined by IFM and a direct analysis for $s/D=3$

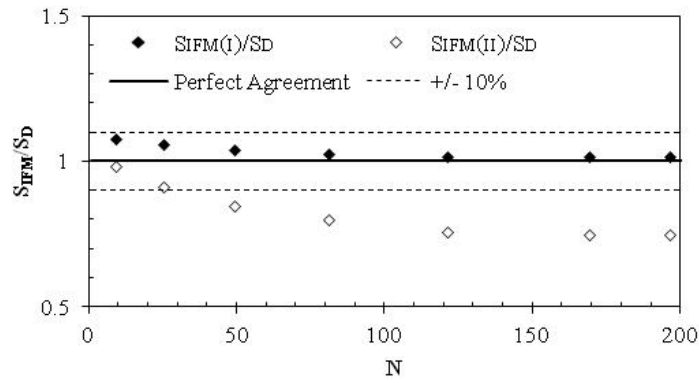


Fig. 14c Comparison between settlement predictions determined by IFM and a direct analysis for $s/D=5$

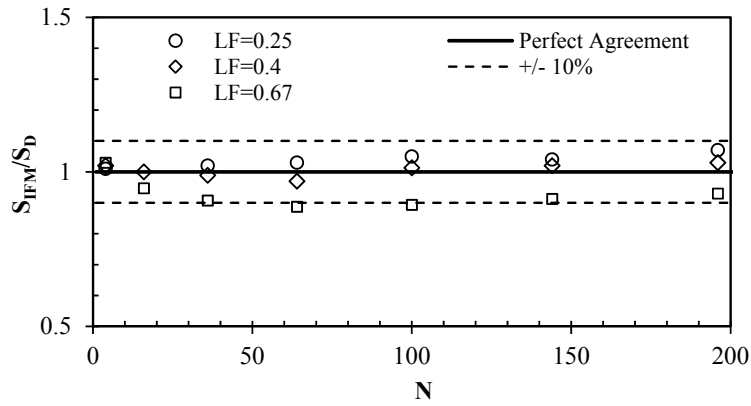


Fig. 14d Influence of load level on accuracy of Approach I; $s/D=3$

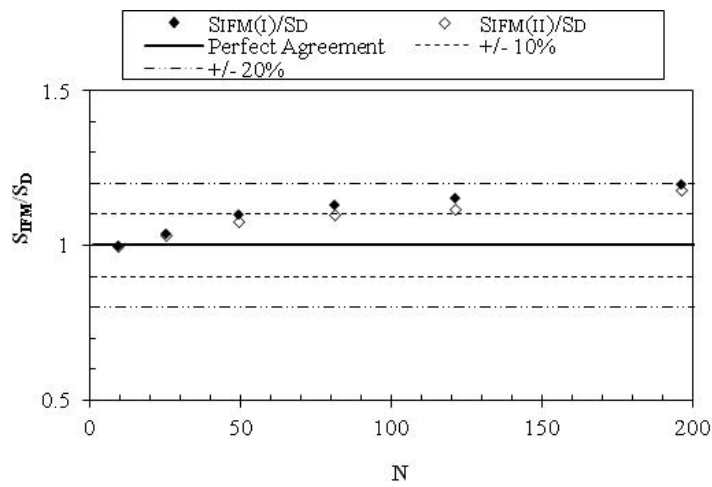


Fig. 15a Comparison between settlement predictions determined by IFM and a direct analysis for $E_B/E_S=10$

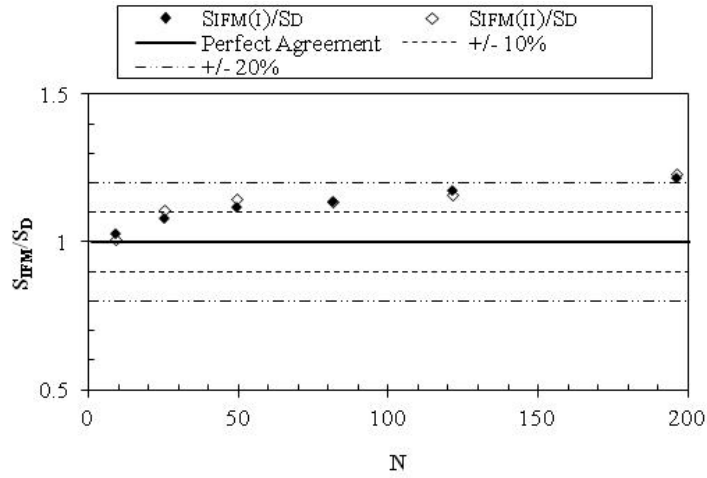


Fig. 15b Comparison between settlement predictions determined by IFM and a direct analysis for $E_B/E_S=30$

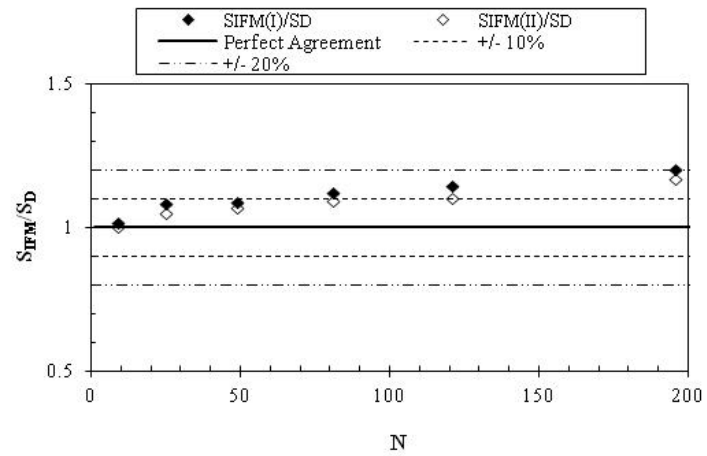


Fig. 15c Comparison between settlement predictions determined by IFM and a direct analysis for $E_B/E_S=50$

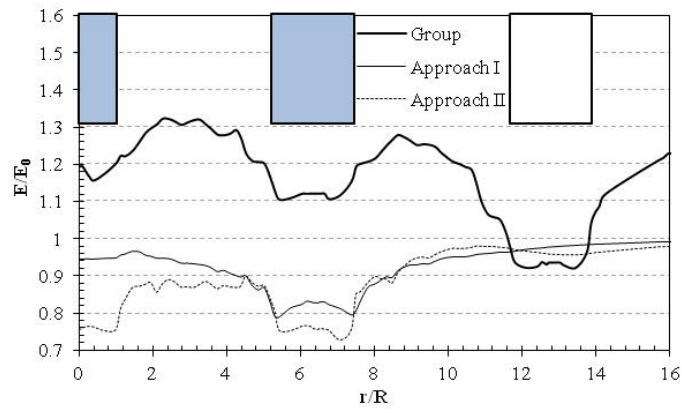


Fig. 16 Comparison of predicted variations in E/E_0 directly beneath the pile base; $LF=0.4$, $N=25$,
 $(s/D)_g=3$, $(s/D)_{IFM}=3$