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Many-valued judgment aggregation: characterizing the possibility/impossibility boundary for an important class of agendas*

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Abstract

A general model of judgment aggregation is presented in which judgments on propositions are not binary but come in degrees. The primitives of the model are a set of propositions, an entailment relation, and a “triangular norm” which establishes a lower bound on the degree to which a proposition is true whenever it is entailed by a set of propositions. For an important class of agendas, we identify a necessary and sufficient condition for judgment aggregation to be free from veto power. This condition says that the triangular norm used to establish the lower bound must contain a zero divisor.

JEL classification: D71.

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1 Introduction

A large literature on judgment aggregation exists, motivated by List and Pettit’s [10] initial contribution. List and Puppe [11] is a survey to which

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we refer the interested reader. The literature is concerned with aggregating profiles of individual judgment sets into a collective judgment set. These judgment sets contain propositions that individuals believe to be true. Since propositions are logically interconnected, a judgment set must contain $p \wedge q$ if it contains both p and q . As is well-known, a collective judgment set can violate this requirement if majority voting is applied to each of these propositions. The theory of judgment aggregation has been developed in response to this observation.

Although this example appears similar to the standard majority voting paradox, the latter is actually a special case of one of these so-called “discursive dilemmas”.¹ The early literature established impossibility results, but more recent work has generated deeper, characterization theorems. For instance, the set of agendas² over which any independent and monotonic judgment aggregation rule is dictatorial has been characterized.³ This result, due to Nehring and Puppe [13], describes the logical structure of agendas giving rise to dictatorship.⁴ In the language of their papers, these agendas are “totally blocked”.

In this paper we restrict attention to this set of agendas but show that relaxing the assumption that each proposition is either true or false can break the Nehring-Puppe theorem (but not always).⁵ We present a simple condition that describes when (and only when) this happens. This condition is expressed within a general model of judgment aggregation, which takes its inspiration from Dietrich’s [2] idea of “general logic”. The model is parsimonious, so that attention can be focused on the two technical concepts that feature in the characterization theorem (triangular norms and zero divisors).

Three features of the model are worth highlighting. First of all, we employ an entailment relation between sets of propositions and propositions. This relation determines what (if any) logical connections exist between elements of the agenda. Secondly, we assume that judgments are deductively closed: if A entails p , then a triangular norm establishes a lower bound on the degree to which p is true. Thirdly, propositions can be neither true nor false (i.e. truth

¹Dietrich and List [3].

²An agenda is a set of propositions over which judgments are made.

³Independence and monotonicity inherit their standard meanings from social choice theory. Majority voting on each proposition is an example of an independent and monotonic rule.

⁴See also Dokow and Holzman [5] and Dietrich and List [4].

⁵We are speaking somewhat loosely here. For one thing, we assume unanimity and not monotonicity. Our dictatorship concept also differs (it is weaker). We also have to translate what Nehring and Puppe mean by “totally blocked” into our framework. However, nothing of significance is affected by this.

is many-valued).⁶ This assumption gives us some formal generality, but it can be defended on other grounds. Many-valued models potentially allow us to “smooth” the aggregation of individual judgments, and so possibility results may arise more naturally.⁷ As we demonstrate in this paper, sometimes this is true and sometimes it is false.⁸ But we can go further than this and actually locate the possibility/impossibility boundary in our model. This is what we accomplish in this paper.

Our characterizing condition is surprisingly simple. Deductive closure says that if a set of propositions A entails proposition p , then a triangular norm establishes a lower bound on the degree to which p is true. Each element of A is true to some degree, and the triangular norm is applied to this set of values. Judgment aggregation is free from veto power if and only if this triangular norm contains a zero divisor. This means that if A entails p , then p could be regarded as false even if no element of A is.

2 Model

Following Dietrich [2], let L denote a non-empty, finite set of formal expressions⁹, called propositions. Let $\mathcal{P}(L)$ denote the set of non-empty subsets of L .

Definition 1. A valuation is a function $f : L \rightarrow [0, 1]$.

Let \models denote a relation on $\mathcal{P}(L) \times L$ such that the following are true for all $A, B \subseteq L$ and all $p \in L$.

- (i) If $p \in A$ then $A \models p$.
- (ii) If $A \models b$ for all $b \in B$, and $B \models p$, then $A \models p$.

The relation \models is an entailment relation, where $A \models p$ is read “ A entails p ”. Condition (i) is a stronger version of Dietrich’s “self-entailment” condition and condition (ii) is what Dietrich calls (quite naturally) “transitivity”.¹⁰

Imagine that $A \models p$ with $A = \{p_1, p_2\}$. To motivate the idea of a triangular norm, consider the relationship between $f(p_1)$, $f(p_2)$ and $f(p)$.

⁶The set of possible values is $[0, 1]$.

⁷Many-valued models are discussed in Salles [16] and Piggins and Salles [15].

⁸This is in keeping with the findings of Dietrich [1], Pauly and van Hees [14], van Hees [8], Dietrich and List [4] and Dokow and Holzman [6].

⁹A formal expression is a finite concatenation of symbols, such as p or $p \rightarrow q$ or $\exists x(Px$ or $s+)) = \odot$.

¹⁰This notion of logical entailment is quite general. Our conditions do not imply “completeness”, nor do they imply “non-paraconsistency”. Dietrich [2] discusses these concepts.

One natural idea is that $\min(f(p_1), f(p_2)) \leq f(p)$. This condition has the virtue of being classically consistent in the sense that if truth-values are restricted to $\{0, 1\}$ and every proposition in A is true, then p is true. Similarly, in this case, if a proposition in A is false, then p is either true or false.

However, as the reader will verify, classical consistency is also obtained with $f(p_1) \times f(p_2) \leq f(p)$. Another case is $\max(f(p_1) + f(p_2) - 1, 0) \leq f(p)$.

All of these functions are triangular norms.¹¹ In our model they establish a lower bound on the degree to which p is true whenever A entails p .

Definition 2. A triangular norm (t-norm) T is a function from $[0, 1]^2$ to $[0, 1]$ such that for all $x, y, z \in [0, 1]$,

- (i) $T(x, y) = T(y, x)$,
- (ii) $T(x, T(y, z)) = T(T(x, y), z)$,
- (iii) $T(x, y) \leq T(x, z)$ if $y \leq z$,
- (iv) $T(x, 1) = x$.

The minimum t-norm is defined as $T_M(x, y) = \min(x, y)$. The product t-norm is defined as $T_P(x, y) = x \times y$. The Lukasiewicz t-norm is defined as $T_L(x, y) = \max(x + y - 1, 0)$.

Definition 3. A t-norm T has no zero divisor if and only if, for all $x, y \in [0, 1]$, $T(x, y) = 0$ implies $x = 0$ or $y = 0$.

Clearly $T_M(x, y)$ has no zero divisor as does $T_P(x, y)$, whereas $T_L(x, y)$ has zero divisors (since $T_L(\frac{1}{2}, \frac{1}{2}) = 0$, for instance). This difference turns out to be crucial.

The associativity of T (condition (ii)) allows us to extend each t-norm in a unique way to an n -ary operation by induction, defining for each n -tuple $(x_1, \dots, x_n) \in [0, 1]^n$,

$$\mathbb{T}_{i=1}^n x_i = T(\mathbb{T}_{i=1}^{n-1} x_i, x_n) = T(x_1, \dots, x_n).$$

Given any valuation f and any $A \in P(L)$ with $A = \{p_1, \dots, p_m\}$ and $m \geq 2$, let $T(f(p_1), \dots, f(p_m))$ be denoted by $f(A)$.

In keeping with the discussion above, we require that individual and collective judgments satisfy the following condition.

Deductive closure. For all $p \in L$ and all $A \in \mathcal{P}(L)$, $A \models p$ implies $f(A) \leq f(p)$.

¹¹A comprehensive reference on t-norms and their applications is Klement, Mesiar and Pap [9]. An application of these concepts to conventional preference aggregation can be found in Duddy, Perote-Peña and Piggins [7].

Let V denote the set of all deductively closed valuations.

We now describe our agenda condition.

Let \triangleright denote a relation on $L \times L$ defined as follows. For all $p, q \in L$, $p \triangleright q$ if there exists a subset $A \subseteq L$ such that $(A \cup \{p\}) \models q$ while $A \not\models q$ or $A = \emptyset$.

Totally blocked. For all $p, q \in L$, there exists a sequence of zero or more propositions p_1, p_2, \dots, p_m such that $p \triangleright p_1 \triangleright p_2 \triangleright \dots \triangleright p_m \triangleright q$.

A slightly stronger condition is the following.

Non-trivially, totally blocked. L is totally blocked and there exists $p, q \in L$ such that $p \triangleright q$ and $\{p\} \not\models q$.

Here is an example of a non-trivially, totally blocked agenda. Let $L = \{a, b, c\}$ and $\{c\} \models a$, $\{c\} \models b$, $\{a, b\} \models c$, $\{b\} \not\models c$ and $\{a\} \not\models c$. Note that this corresponds to the agenda p, q and $p \wedge q$ in propositional logic, with $a = p, b = q$ and $c = p \wedge q$.

Let N denote a finite set of individuals with $\#N = n \geq 2$. The individuals in N are ordered (and “labelled”) from 1 to n .

An aggregation rule is a function $\Phi : V^n \rightarrow V$. Let ϕ denote $\Phi(v_1, \dots, v_n)$, ϕ' denote $\Phi(v'_1, \dots, v'_n)$ and so on.

The following are properties that aggregation rules may satisfy.¹²

Independence. For all $p \in L$ and all $(v_1, \dots, v_n), (v'_1, \dots, v'_n) \in V^n$, $v_i(p) = v'_i(p)$ for all $i \in N$ implies $\phi(p) = \phi'(p)$.

Systematicity. For all $p, q \in L$ and all $(v_1, \dots, v_n), (v'_1, \dots, v'_n) \in V^n$, $v_i(p) = v'_i(q)$ for all $i \in N$ implies $\phi(p) = \phi'(q)$.

Unanimity. For all $v \in V$, $\Phi(v, \dots, v) = v$.

Veto-dictatorial. There exists some $i \in N$ such that for all $p \in L$ and every $(v_1, \dots, v_n) \in V^n$, $v_i(p) = 0$ implies $\phi(p) = 0$.

3 Theorem

The following lemma derives systematicity from independence and unanimity, as is common in the literature.¹³

Lemma. *Assume that L is totally blocked and that Φ is independent and unanimous. Then Φ is systematic.*

¹²Mongin [12] is an important discussion of the properties of aggregation rules.

¹³See, in particular, Theorem 2 in Dietrich and List [4]. However, since our models differ it is important to provide an independent statement and proof of this result.

Proof. Take a pair of propositions $p, q \in L$ such that $p \triangleright q$. By definition, it must be true that $\{p\} \vDash q$ or that there exists a non-empty set $A \subseteq L$ such that $(A \cup \{p\}) \vDash q$ while $A \not\vDash q$.

Case 1. Suppose that $\{p\} \vDash q$. It follows that for any $(v_1, \dots, v_n) \in V$, $\phi(p) \leq \phi(q)$ since ϕ is in V .

Case 2. Suppose that there exists a non-empty set $A \subseteq L$ such that $(A \cup \{p\}) \vDash q$ while $A \not\vDash q$. Take any profile $(v_1, \dots, v_n) \in V^n$ such that $v_i(p) = v_i(q)$ for all $i \in N$. There exists a profile $(v'_1, \dots, v'_n) \in V^n$ with $v'_i(p) = v'_i(q) = v_i(p)$ and $v'_i(a) = 1$ for all $a \in A$ and all $i \in N$.¹⁴ Unanimity and independence imply that $\phi'(a) = 1$ for all $a \in A$. Recalling property (iv) of a t-norm, note that $\phi'(A) = 1$ implies that $T(\phi'(A), \phi'(p)) = \phi'(p)$. Since $(A \cup \{p\}) \vDash q$, we know that $T(\phi'(A), \phi'(p)) \leq \phi'(q)$ and so $\phi'(p) \leq \phi'(q)$. It must also be true, since Φ is independent, that $\phi(p) \leq \phi(q)$.

We can use this argument to show that Φ is systematic.

Since the agenda is totally blocked we know that for all $r, s \in L$ either $r \triangleright s$ or there exists $p_1, \dots, p_k \in L$ such that $r \triangleright p_1 \triangleright \dots \triangleright p_k \triangleright s$. By applying the above argument repeatedly we can see that for any profile $(v''_1, \dots, v''_n) \in V^n$ such that $v''_i(r) = v''_i(s)$ for all $i \in N$, it must be the case that $\phi''(r) \leq \phi''(s)$. Similarly, either $s \triangleright r$ or the agenda must contain q_1, \dots, q_k such that $s \triangleright q_1 \triangleright \dots \triangleright q_k \triangleright r$ and so $\phi''(s) \leq \phi''(r)$. Hence $\phi''(r) = \phi''(s)$. \square

We are now in a position to state our theorem.

Let \mathcal{H} denote the set of all aggregation rules that are independent, unanimous and not veto-dictatorial.

Theorem. *Assume that L is non-trivially, totally blocked. Then \mathcal{H} is empty if and only if T has no zero divisor.*

Proof. We prove that \mathcal{H} is empty if T has no zero divisor.

Take any $p, q \in L$ such that $p \triangleright q$ and $\{p\} \not\vDash q$. There must exist a non-empty set $A \subseteq L$ such that $(A \cup \{p\}) \vDash q$ while $B \not\vDash q$ for all non-empty $B \subsetneq A \cup \{p\}$. Let r denote an element of A such that $\{p\} \not\vDash r$ and let Z denote $A - \{r\}$. If Z is empty then ignore the references to it in the tables below.

Take a profile $(v_1, \dots, v_n) \in V^n$ such that $v_i(p) = 0$ for all $i \in N$. Unanimity and independence imply that $v(p) = 0$. Take some other profile $(\hat{v}_1, \dots, \hat{v}_n) \in V^n$ such that $\hat{v}_i(p) = 1$ for all $i \in N$. By identical logic,

¹⁴This profile might not exist if, for example, $q \in A$ or $A \vDash p$, or if there exists some $r \in L$ such that $A \vDash r$ and $\{r\} \vDash q$. However, these and other possibilities are excluded by properties (i) and (ii) of the entailment relation.

$\hat{v}(p) = 1$. Consider the following sequence of profiles.

$$\begin{aligned}\mathbf{W}^{(0)} &= (v_1, \dots, v_n), \\ \mathbf{W}^{(1)} &= (\hat{v}_1, v_2, \dots, v_n), \\ \mathbf{W}^{(2)} &= (\hat{v}_1, \hat{v}_2, v_3, \dots, v_n), \\ &\dots \\ \mathbf{W}^{(n)} &= (\hat{v}_1, \dots, \hat{v}_n).\end{aligned}$$

Let $\phi^{(0)}$ denote $\Phi(\mathbf{W}^{(0)})$, $\phi^{(1)}$ denote $\Phi(\mathbf{W}^{(1)})$ and so on. At some profile in this sequence the value assigned to p by the collective valuation must rise from zero to a value strictly greater than zero. Assume, without loss of generality, that this happens at $\mathbf{W}^{(2)}$.

We can construct a profile $(v'_1, \dots, v'_n) \in V^n$ where individuals assign the following values.

	p	q	r	Z
Individual 1	1	1	1	1
Individual 2	0	0	1	1
Everyone else	1	0	0	1

Systematicity implies that $\phi'(r) = \phi^{(2)}(p)$, and that $\phi'(q) = \phi^{(1)}(p)$. Property (iv) of T implies that $\phi'(A) = \phi'(r)$, and so $\phi'(A) > 0$. Recall that $(A \cup \{p\}) \models q$ and so $T(\phi'(A), \phi'(p)) \leq \phi'(q)$. Since $\phi'(q) = 0$ it must be the case that $T(\phi'(A), \phi'(p)) = 0$. Given that $\phi'(A) > 0$ and T has no zero divisor, it must be the case that $\phi'(p) = 0$.

We have seen that $\phi'(p) = 0$ despite the fact that $v'_i(p) = 1$ for all $i \in N - \{2\}$. The proof of veto-dictatorship can be completed as follows.

Either it is true or false that $q \triangleright p$, and so we examine two cases. Let (t_1, \dots, t_n) be any element of $[0, 1]^n$.

Case 1. Assume it is false that $q \triangleright p$. Consider a profile $(v''_1, \dots, v''_n) \in V^n$ where individuals assign the following values.

	p	q	r	Z
Individual 1	t_1	1	1	1
Individual 2	0	0	1	1
Individual 3	t_3	0	0	1
...				
Individual n	t_n	0	0	1

Systematicity implies that $\phi''(r) = \phi^{(2)}(p)$ and that $\phi''(q) = \phi'(q)$. Property (iv) of a t-norm implies that $\phi''(A) = \phi''(r)$, and so $\phi''(A) > 0$. Since $(A \cup \{p\}) \vDash \{q\}$ we have $T(\phi''(A), \phi''(p)) \leq \phi''(q)$. As T has no zero divisor, it must be the case that $\phi''(p) = 0$.

Case 2. Assume $q \triangleright p$. It must be true that there exists a subset $C \subseteq L$ such that $(C \cup q) \vDash p$ while $C \not\vDash p$ or $C = \emptyset$. If C is empty then ignore the references to it in the table below. Consider a profile $(v_1^*, \dots, v_n^*) \in V^n$ where individuals assign the following values.

	p	q	C
Individual 1	1	t_1	1
Individual 2	0	0	1
Individual 3	1	t_3	1
...			
Individual n	1	t_n	1

Independence implies that $\phi^*(p) = \phi'(p) = 0$. Additionally, due to property (iv) of a t-norm or the fact that C is empty, we know that $\phi^*(C \cup q) = \phi^*(q)$. Since $(C \cup q) \vDash p$, it must be true that $\phi^*(C \cup q) \leq \phi^*(p)$ and so $\phi^*(q) \leq \phi^*(p)$. In other words, $\phi^*(q) = 0$.

The desired result follows from systematicity.

We prove that \mathcal{H} is not empty if T has a zero divisor. First, note that if T has a zero divisor then, since T satisfies property (iii), there must exist $z \in (0, 1)$ such that $T(z, z) = 0$.

Define an aggregation rule Ψ as follows. For all $(v_1, \dots, v_n) \in V^n$ and all $p \in L$, let $\psi(p)$ be equal to the median of the three numbers $z, \max(v_1(p), \dots, v_n(p))$

and $\min(v_1(p), \dots, v_n(p))$. It is clear that Ψ is independent and unanimous. We need to prove that for all $(v_1, \dots, v_n) \in V^n$, ψ is deductively closed.

Assume, by way of contradiction, that ψ is not deductively closed. Then there exists $p \in L$, $A \in \mathcal{P}(L)$ and $(v_1, \dots, v_n) \in V^n$ such that $A \models p$ and $\psi(A) > \psi(p)$.

Let $Z = \{a \in A \text{ such that } \psi(a) > z\}$. If $Z = \emptyset$ then $\psi(a) \leq z$ for all $a \in A$, and so $\psi(A) = T(\psi(a_1), \dots, \psi(a_m)) \leq T(z, \dots, z)$ by property (iii) of a t-norm. Since $T(z, z) = 0$ then $\psi(A) = 0$ which contradicts $\psi(A) > \psi(p)$. Therefore, $Z \neq \emptyset$.

Case 1. $Z = A$. Recalling the definition of Ψ , $\psi(a) > z$ for all $a \in A$ implies that $\psi(a) = \min(v_1(a), \dots, v_n(a))$ for all $a \in A$. Let $j \in N$ be an individual such that $v_j(p) = \min(v_1(p), \dots, v_n(p))$. Since v_j is deductively closed, we know that $v_j(A) \leq v_j(p)$. Of course, it must be the case that $v_j(a) \geq \min(v_1(a), \dots, v_n(a))$ for all $a \in A$, which implies $v_j(a) \geq \psi(a)$ for all $a \in A$. Given property (iii) of a t-norm, it must be the case that $\psi(A) \leq v_j(A)$. Hence $\psi(A) \leq v_j(p)$. We have $v_j(p) = \min(v_1(p), \dots, v_n(p))$ and we know that $\psi(p)$ cannot be less than $\min(v_1(p), \dots, v_n(p))$, and so we have $\psi(A) \leq \psi(p)$. This contradicts our assumption that $\psi(A) > \psi(p)$.

Case 2. $A - Z$ is a singleton. Given property (iii) of a t-norm, we know that $T(\psi(Z), z) \geq \psi(A)$. Therefore, given our initial assumption that $\psi(A) > \psi(p)$, it must be the case that $T(\psi(Z), z) > \psi(p)$. We know by property (iv) of a t-norm that $T(1, z) = z$. Given property (iii), we have $T(\psi(Z), z) \leq z$. Since we have $T(\psi(Z), z) > \psi(p)$ and $T(\psi(Z), z) \leq z$, it must be true that $z > \psi(p)$. Returning to the definition of Ψ , note that $z > \psi(p)$ implies $\psi(p) = \max(v_1(p), \dots, v_n(p))$. Note too that $\psi(b) > z$ implies $\psi(b) = \min(v_1(b), \dots, v_n(b))$ for all $b \in Z$. We know then that for all $i \in N$ and all $b \in Z$, $v_i(b) \geq \psi(b)$ and $v_i(p) \leq \psi(p)$. So, letting $\{c\} = A - Z$, there must exist an individual $k \in N$ such that for all $b \in Z$, $v_k(b) \geq \psi(b)$, $v_k(p) \leq \psi(p)$ and $v_k(c) = \max(v_1(c), \dots, v_n(c))$. Since v_k is deductively closed, we know that $T(v_k(Z), v_k(c)) \leq v_k(p)$. We know by the definition of Ψ that $\psi(c) \leq \max(v_1(c), \dots, v_n(c))$, and so we have $\psi(c) \leq v_k(c)$. Given property (iii) of a t-norm and $\psi(Z) \leq v_k(Z)$ and $\psi(c) \leq v_k(c)$, it must be true that $\psi(A) \leq v_k(A)$. Hence $\psi(A) \leq v_k(p)$ and so, since $v_k(p) \leq \psi(p)$, we have $\psi(A) \leq \psi(p)$. However, this contradicts our assumption that $\psi(A) > \psi(p)$.

Case 3. The cardinality of $A - Z$ is greater than one. Given property (iii) of a t-norm and since $T(z, z) = 0$, we know that $\psi(A - Z) = 0$. Property (iv) implies that $T(1, 0) = 0$ and so, by property (iii), we know that $T(x, 0) = 0$ for all $x \in [0, 1]$. Hence $\psi(A) = 0$ and it is impossible that $\psi(A) > \psi(p)$. \square

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