



## **A nested version of the combined hydrodynamic and water quality model, DIVAST**

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# A Nested Version of the Combined Hydrodynamic and Water Quality Model, DIVAST

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## Abstract

The present research deals with the development of a one-way nested version of the combined hydrodynamic and water quality model DIVAST (Depth Integrated Velocity and Solute Transport). Initial research has concentrated on the development of a nested version of the hydrodynamic module of the model. Details of the nesting procedure used in the research are presented. The interpolation techniques employed at the fine/coarse grid boundaries are discussed as this is a vital element of any grid nesting scheme. Results obtained from the nested hydrodynamic model are presented and discussed. Comparisons are made between results from the nested model and coarse and fine resolution models of the same model domain. The results demonstrate the numerical accuracy of the nested model and the benefits of suitable boundary interpolation techniques.

**Keywords:** nested model, hydrodynamics, interpolation, one-way nesting, DIVAST.

## 1 Introduction

One of the most common problems in hydrodynamic and water quality modelling is the location of open boundaries; they must be located such that their conditions will not adversely affect model predictions in the region of interest. This problem often leads to a situation which requires a large computational domain, of which the area of interest comprises only a small percentage. If a finite difference model is applied, the associated orthogonal, finite difference grid can become very large, particularly if a high spatial resolution is required in the area of interest. In addition a higher spatial resolution requires a higher temporal resolution. This may result in an excessively high computational cost. One common solution to this problem is the use of a dynamic nesting method. This method allows one to increase spatial resolution in a sub-region of the model domain without incurring the computational expense of fine resolution over the entire domain.

The Author is currently involved in a modelling study which demonstrates this problem. A validated water quality model [1], based on the DIVSAT model, has been developed to simulate nutrient dynamics and primary production in Cork Harbour, a waterbody on the southwest coast of Ireland. The bathymetrical model of the harbour is shown in Figure 1. In order to predict localised phytoplankton blooms in the bays, loughs and channels within the harbour, to a sufficient degree of accuracy, a fine spatial resolution of 30m was required. However, it can be seen that the domain includes large areas of open water where such a high resolution is unnecessary.

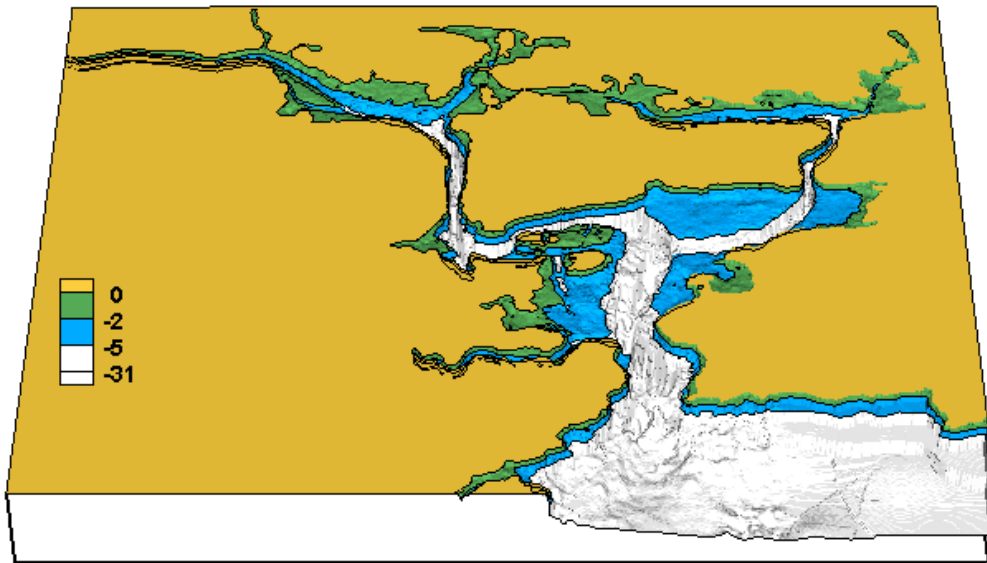


Figure 1: Bathymetrical model of Cork Harbour.

DIVAST employs a two-dimensional, orthogonal, finite difference grid. The fine 30m resolution required for the Cork Harbour model combined with the domain dimensions of approximately 21km x 17km result in a finite difference grid of almost 400,000 cells. The primary production cycle within the model is one of the more complex available and involves the simulation of eight water quality parameters including nutrients, dissolved oxygen and chlorophyll\_a. In addition, the scale of a phytoplankton simulation period is of the order of weeks/months. The computational cost is therefore very large, with typical model run-times of the order of weeks. In an effort to reduce the computational cost it was decided to develop a generic nested grid version of the DIVAST model which could be employed in the Cork model.

Nested models fall into two categories, one-way (passive) and two-way (interactive). One-way models use boundary conditions for the high resolution region that have been obtained from a low resolution calculation. This class of models is also known as passive because the coarse resolution flow field affects the fine resolution region by providing boundary conditions for the fine grid domain,

however, there is no mechanism by which the evolution in the fine resolution region can affect the flow field in the coarse grid (and hence its own boundary conditions). Interactive models, in addition to providing boundary conditions for the fine grid region, allow the evolution within the fine grid to influence the evolution on the coarse grid. Although there are advantages to the interactive system they are necessarily more complicated and computationally expensive [2]. A one-way model approach was therefore deemed best for this research.

In order to simplify the development of the nested model a simple rectangular harbour model was used rather than the Cork model with its more complex bathymetry and hydrodynamics. In addition, the rectangular harbour was designed so that flooding and drying of grid cells did not occur within the domain as flooding and drying complicates the nesting procedure.

## 2 Model Equations

The governing differential equations used by the model to determine the water surface elevation and depth-integrated velocity fields in a horizontal plane are based on integrating the three-dimensional Navier-Stokes equations over the water depth. This results in a two-dimensional model which resolves variables in two mutually perpendicular horizontal directions ( $x$  and  $y$ ). The depth-integrated continuity and  $x$ -direction momentum equations can be shown to be given by equations (1) and (2) respectively [3]:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = 0 \quad (1)$$

$$\frac{\partial q_x}{\partial t} + \beta \left[ \frac{\partial U q_x}{\partial x} + \frac{\partial V q_y}{\partial y} \right] = \quad (2)$$

$$f q_y - g H \frac{\partial \zeta}{\partial x} + \frac{\tau_{xw}}{\rho} + \frac{\tau_{xb}}{\rho} + 2 \frac{\partial}{\partial x} \left[ \epsilon H \frac{\partial U}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \epsilon H \left[ \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right] \right]$$

where,  $\zeta$  = water surface elevation above mean water level  
 $t$  = time  
 $q_x, q_y$  = depth-integrated velocity flux components in the  $x, y$  directions  
 $\beta$  = momentum correction factor  
 $U, V$  = depth-integrated velocity components in the  $x, y$  directions  
 $f$  = Coriolis parameter  
 $g$  = gravitational acceleration  
 $H$  = total depth of water column  
 $\tau_{xw}$  = surface wind shear stress components in the  $x$  direction  
 $\tau_{xb}$  = bed shear stress component in the  $x$  direction  
 $\rho$  = fluid density  
 $\epsilon$  = depth averaged turbulent eddy viscosity

The finite difference scheme used in the model is based upon the Alternating Direction Implicit (ADI) technique. This involves the sub-division of each timestep into two timesteps allowing a two-dimensional implicit scheme to be applied but considering only one dimension implicitly for each half-timestep, without the solution of a full two-dimensional matrix [3]. The solution scheme proceeds in the x-direction during the first half-timestep computing the water surface elevation,  $\zeta$ , and the x-direction velocity component,  $U$ . It then proceeds in the y-direction during the second half-timestep computing the water elevation and the y-direction velocity component,  $V$ . A space-staggered grid system is used with water elevation located at the grid centre and with velocity components,  $U$  and  $V$ , located at the centre of the grid sides. Water depths are also specified at the centre of the grid sides.

### 3 Nesting Procedure and Model Application

The nested model developed in this research actually involves two model simulations running simultaneously. The main run works on the complete model domain within which the grid resolution is relatively coarse. The secondary model works on a much smaller area comprising one or more areas of local interest. The numerical grid for this run can therefore be much finer than the grid of the main simulation. The conditions at the open boundaries of the nested domains are obtained by interpolating results from the coarse model.

A hydrodynamic model of an artificial rectangular harbour was used to develop and test the nested model. The implementation of the nesting procedure in the model allows the specification of any integer ratio between the size of the cells in the coarse model and their size in the nested model. A nesting ratio of 3:1 was used in this study. For an odd grid ratio the variable values in a coarse grid point will always match those in the fine grid allowing inter-comparison between coarse and fine grid points. Any number of nested grids may be specified but only one was used for model development. A schematic of the nested model domain is shown in Figure 2.

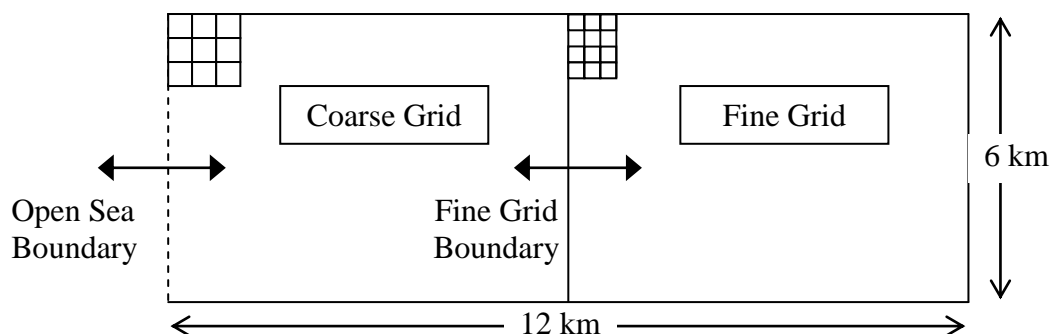


Figure 2: Schematic of nested grid in rectangular harbour model.

The nested model hydrodynamics are driven by the water surface elevations calculated by the coarse model along the fine mesh open boundaries. When

transferring the water elevation data from the coarse grid to the fine grid boundary an interpolation procedure is required since the number of fine grid points along the boundary is greater than the number of coarse grid points. In addition the fine-grid model requires a higher degree of temporal resolution, therefore, data must also be interpolated with respect to time to cater for the difference in model timesteps and obtain a complete set of boundary data.

### 3.1 Spatial Interpolation

In this study, three different methodologies were tested for specifying boundary conditions to the nested domain. These included 1) a zeroth-order interpolation scheme, 2) a linear interpolation scheme, and 3) a quadratic interpolation scheme, all of which are commonly used interpolation schemes. An intercomparison of model results obtained for each scheme is presented in the results section. Brief descriptions of the three schemes are now presented.

#### 3.1.1 Zeroth-order Interpolation Scheme

The assumption of uniform distribution of the scalar within the grid cell is the basis for the zeroth-order interpolation scheme. In this scheme, all nested grid cells that lie within a particular coarse grid cell are assigned the same value as that of the coarse grid cell. If  $\zeta_i$  represents the water elevation for a coarse grid cell  $i$ , then the elevation  $\zeta_k$  for all the fine grid cells within its confines can be written as:

$$\zeta_k = \zeta_i, \quad k = 1, \dots, m \quad (3)$$

where  $m$  is the number of fine grid cells within the coarse grid cell.

#### 3.1.2 Linear Interpolation Scheme

The linear interpolation scheme assumes a linear relationship between the scalar values of adjacent coarse grid cells in a given direction. A proportional coefficient is used to determine the value of the scalar at a fine grid cell within the coarse grid cell. The water elevation  $\zeta$  in a fine grid cell  $k$  which lies within the coarse grid cell  $i$  may be calculated as

$$\zeta_k = \zeta_{i-1} + \omega(\zeta_i - \zeta_{i-1}) \quad (4)$$

with  $\omega$ , the proportional interpolation coefficient, further expressed as

$$\omega = \frac{\Delta X + (2k - 1)\Delta x}{2\Delta X}, \quad k = 1, \dots, n. \quad (5)$$

where  $n$  is the nesting ratio, such that  $n\Delta x = \Delta X$ , where  $\Delta x$  and  $\Delta X$  represent the fine- and coarse-grid sizes, respectively.

### 3.1.3 Quadratic Interpolation Scheme

The quadratic interpolation scheme is based on the conservation formula for nested grids suggested by Kurihara et al. [4] and Clark and Farley [5]. For a given nesting ratio  $n$ , the interpolation of the coarse grid water elevation  $\zeta_i$  to the fine grid cells within it may be represented as [6]

$$\zeta_k = E_{i-1}^k \zeta_{i-1} + E_i^k \zeta_i + E_{i+1}^k \zeta_{i+1} \quad (6)$$

with the function  $E$  further expressed as

$$E_{i-1}^k = \frac{\varepsilon_k (\varepsilon_k - 1)}{2} + \alpha \quad (7)$$

$$E_i^k = (1 - \varepsilon_k^2) - 2\alpha \quad (8)$$

$$E_{i+1}^k = \frac{\varepsilon_k (\varepsilon_k + 1)}{2} + \alpha \quad (9)$$

where  $\varepsilon$  represents a normalised local coordinate pointing in the same direction as the global coordinate and whose origin coincides with the centre of the coarse grid cell  $i$ . The value of  $\varepsilon$  for the fine grid cell  $k$  is defined as:

$$\varepsilon_k = \frac{(2k-1)\Delta x - \Delta X}{2\Delta X}, \quad k = 1, \dots, n \quad (10)$$

and

$$\alpha = \frac{1}{24} \left[ \left( \frac{\Delta x}{\Delta X} \right)^2 - 1 \right] \quad (11)$$

The parameter  $\alpha$  is introduced to ensure mass conservation as discussed by Clark and Farley [5].

## 3.2 Temporal Interpolation

The temporal resolution of a hydrodynamic model is based upon the Courant Number,  $C_r$ , defined as

$$C_r = \frac{\sqrt{gH} \cdot \Delta t}{\Delta x} \quad (12)$$

where  $H$  is the maximum water depth,  $\Delta t$  is the model timestep,  $\Delta x$  is the grid spacing and  $g$  is gravitational acceleration. Within DIVAST the Courant Number should be less than eight to avoid hydrodynamic instabilities. From equation (3) it is apparent that a reduction in grid spacing must be accompanied by a reduction in

model timestep. In this study, the nested model was run using a coarse-grid timestep of 120 seconds and a fine-grid timestep of 40 seconds, giving a temporal ratio of 3:1. Thus, for each coarse model timestep the fine model must resolve the model equations for three internal timesteps requiring boundary data at each of these timesteps. A linear interpolation procedure similar to that described in the previous section was used to compute nested boundary water elevations for each fine model timestep from coarse model results. Figure 3 shows the time integration order for the nested model. The temporal ratio used in the nested model can have any value which satisfies the Courant condition. However, whole integer values are best as they simplify the interpolation procedure.

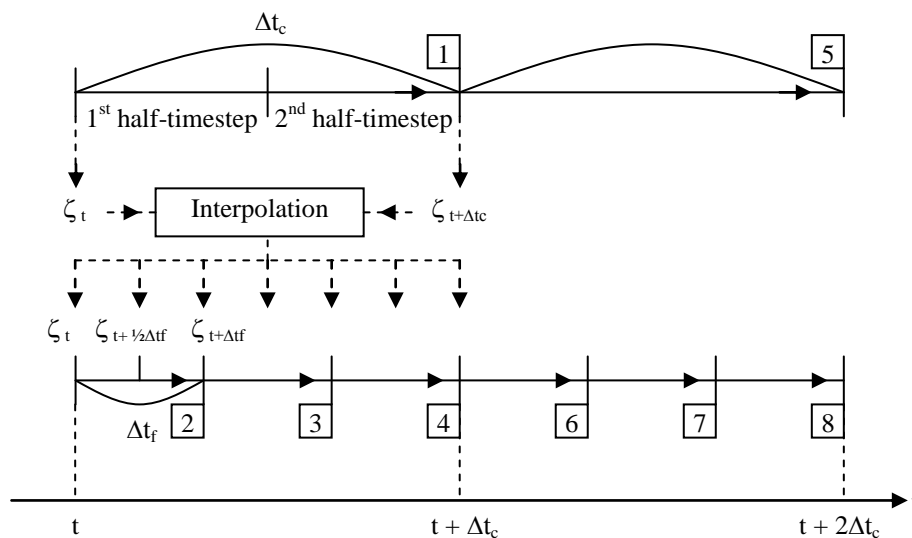


Figure 3: Time integration order for the nested model.

## 4 Results and Discussion

Results are presented from five different models of the artificial rectangular harbour. These included coarse and fine resolution models of the harbour and various versions of the nested model. The nested models differed in the manner in which the boundary conditions for the fine mesh were transferred from the coarse mesh. The spacing and timesteps for the coarse and fine grids within the nested models were the same as those used in the single grid models with the exception of the basic nested model. The basic nested model did not employ time-interpolation of nested boundary parameters; the nested grid was resolved at the same timestep as the coarse-grid model. The models and their details are as follows:

- Model 1: coarse model - 120m spacing; 120s timestep [Coarse]
- Model 2: fine model - 40m spacing; 40s timestep [Fine]
- Model 3: basic nested model - no time interpolation [Basic]  
- zeroth-order spatial interpolation



- Model 4: time-interpolated nested model [Time]
  - linear temporal interpolation
  - zeroth-order spatial interpolation
- Model 5: time- and space-interpolated nested model
  - linear time interpolation
  - linear spatial interpolation [Linear]
  - quadratic spatial interpolation [Quad]

The names in brackets in the above description are used in the intercomparison graphs presented below.

Model accuracy was determined by examining the temporal variation in predicted current velocities and water surface elevations at particular locations (time-traces) and the spatial variation of the same parameters at particular instances in time (snapshots). It was expected that results from the fine-grid model would differ from those of the coarse-grid model as a result of improved accuracy due to the higher spatial and temporal resolution. In turn, it was also expected that predictions within the fine domain of the nested models would agree with those from the fine-grid model. Water surface elevation was not expected to vary significantly between models as it is the parameter specified at the open boundaries. Significant variations were expected for current velocities between the coarse-grid and fine-grid simulations.

Figure 4 compares the water surface elevations predicted by all models at a location in the inner harbour within the nested domain, over the course of a tidal cycle. As can be seen the values appear identical with all of the curves superimposed on one another.

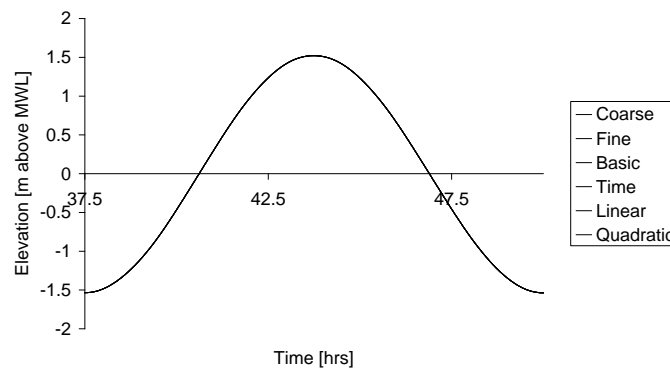


Figure 4: Comparison of water elevations in metres above mean water level (MWL).

Figure 5 shows a contoured plot of the percentage error between the elevations predicted by the fine- and coarse-grid models at mid-flood (40.625 hrs). Also shown is the percentage error calculated between the fine-grid model and the basic nested model at the same stage of tide. The maximum error can be seen to be in the region of 0.3%. Similar comparisons between the other models for all stages of the tide showed that the maximum percentage error in water elevation predictions was

0.32%. It can therefore be concluded that no significant change in water elevation predictions occurred within the model domain for the various models.

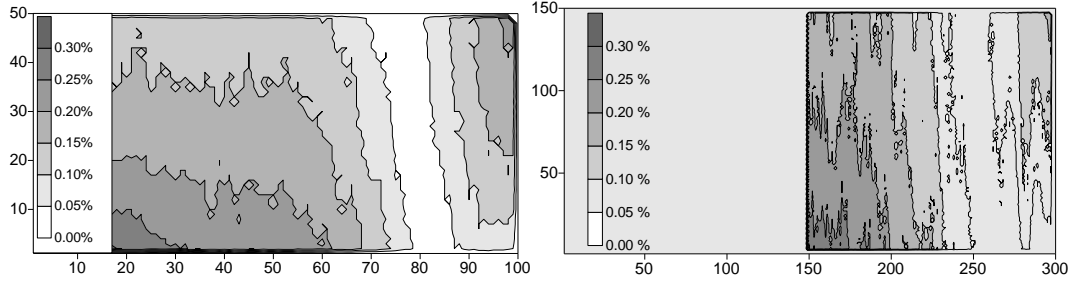


Figure 5: Percentage error in predicted water elevations between coarse- and fine-grid models (left) and fine-grid and basic nested models (right) at mid-flood.

In light of the absence of any significant variation in water elevation with model type, current velocities were examined as a means of determining model performance and the effects of the nested interpolation techniques thereon. Figure 6 compares current velocity magnitudes predicted by the coarse- and fine-grid models at two locations, the first (6a) located half-way into the harbour and the second (6b) located close to the back wall of the harbour. Minimal variation in current velocity is observed at the mid-harbour location. However, at the back of the harbour the computed velocity in the fine-grid model is greater than that in the coarse-grid model over the whole course of the tidal cycle. This point is better demonstrated by Figure 7 which compares the percentage error in current velocities computed by the coarse- and fine-grid models at mid-flood (40.625 hrs) and mid-ebb (43.75 hrs).

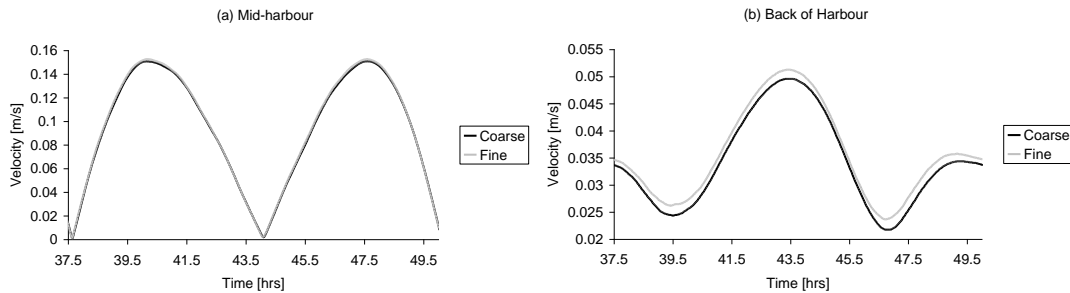


Figure 6: Coarse- and fine-grid velocities at two locations in the harbour.

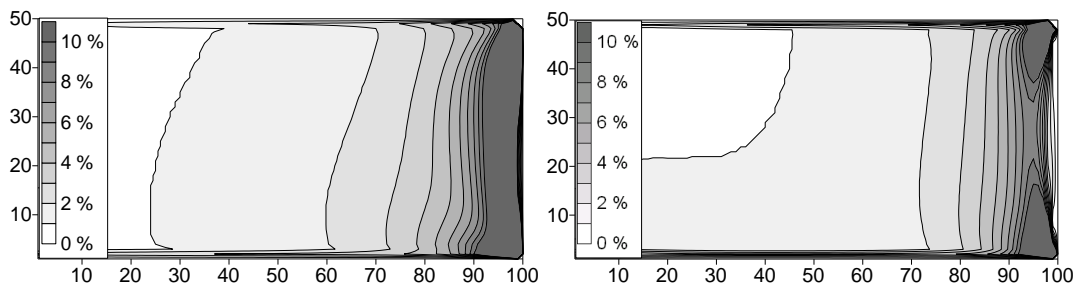


Figure 7: Percentage error in predicted velocities between coarse- and fine-grid models at mid-flood (left) and mid-ebb (right).

It can be seen from Figure 7 that the maximum error in velocity magnitude occurs at the back of the harbour with percentage error between the coarse- and fine-grid predictions reaching up to 10%. It can also be seen that at locations in mid-harbour the percentage error is only in the region of 1-2%. These observations correlate with the time-trace comparisons in Figure 6.

Having established that the higher resolution of the fine-grid model did indeed compute different current velocity results to the coarse-grid model, the various nested models were then compared to the fine-grid model. This comparison could only be carried out within the nested domain. Figure 8 compares current velocities predicted by the fine-grid model with those from all four versions of the nested model at the two locations previously used. Current velocities predicted by all versions of the nested model at mid-harbour (8a), near the nested boundary, matched those predicted by the fine-grid model with the curves superimposed. At the back of the harbour (8b) all nested models, except the basic nested model, predicted similar current velocities to those of the fine-grid model. It is understandable that the basic nested model gave slightly different predictions as it was run using the coarse model timestep of 120 seconds. On the basis of similar time-trace analysis at other locations, it appeared that all nested models, with the exception of the basic version, were as accurate as the fine-grid model.

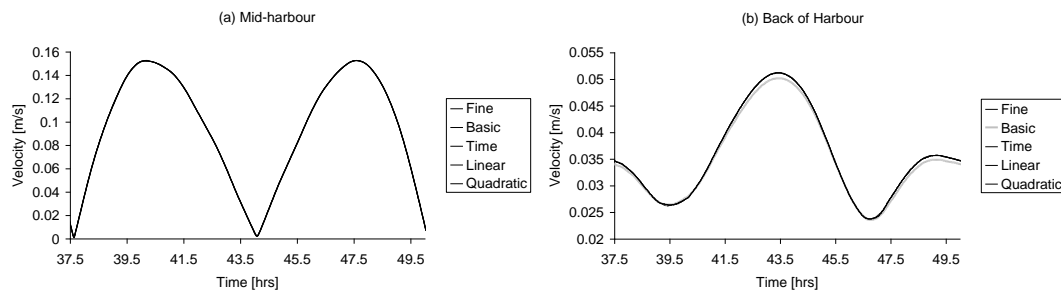


Figure 8: Fine-grid and nested model velocities at two locations in the harbour.

In order to further investigate the accuracy of the nested model and to discern the effect of the boundary interpolation techniques on the nested model, snapshots of velocity magnitudes from the nested models were compared with those from the fine-grid model at various stages of the tide and percentage errors calculated. Figure 9 shows one such set of plots at mid-flood tide (40.625 hrs). It can be seen from the diagrams that the percentage error in the vast majority of the nested domain is less than 2% indicating very good agreement between the nested and fine-grid models. At the back of the harbour, and in particular in the two corners, the error is relatively high due to under-prediction of current velocities by the nested model. However, the velocities in these areas are quite low (less than 0.05 m/s) and small errors in low velocity magnitudes can represent quite significant percentage errors. The actual magnitude of the error in the area is very small as demonstrated in Figure 10. The figure shows the magnitude of error in velocity predictions between the fine-grid and quadratic-interpolated nested model. It can be seen that the maximum error in

the inner harbour is less than 0.004 m/s. The magnitude of this error was found to be independent of the type of spatial interpolation technique employed. The source of the error is thought to be due to a small reduction in mass flux across the nested domain boundary.

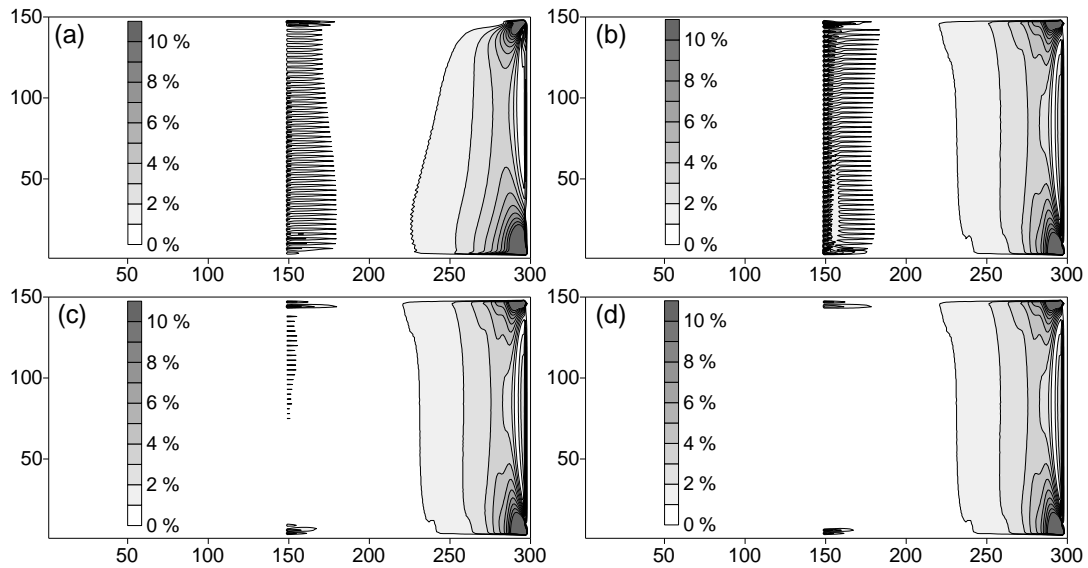


Figure 9: Percentage error in velocities calculated by (a) basic nested, (b) time-interpolated, (c) time- and linear-interpolated, and (d) time- and quadratic-interpolated models compared with fine-grid model.

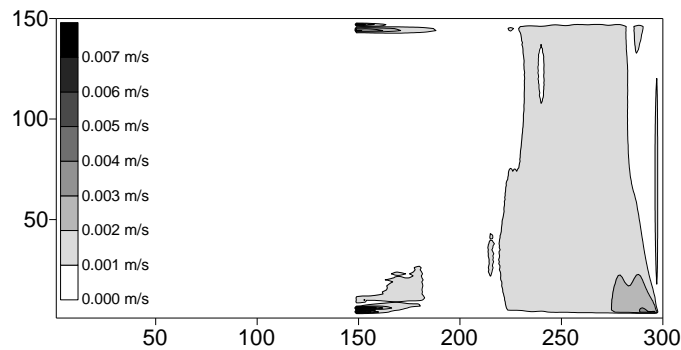


Figure 10: Magnitude of error in current velocity predictions between fine-grid model and quadratic-interpolated nested model.

The most important observation from Figure 9 is the errors that occurred close to the nested boundary. It can be seen that these errors were reduced as the complexity of the spatial interpolation technique was increased. This demonstrates the importance of the procedure for the transfer of boundary conditions between the coarse and fine domains. The use of the zeroth-order interpolation technique caused perturbations in current velocity in the region of the open boundary (9b). Although these perturbations still persisted with the use of the linear interpolation technique (9c), they were significantly diminished in magnitude. The variations were almost completely eliminated by the use of the quadratic interpolation technique (9d).

Errors still persisted close to the top and bottom banks of the harbour, however, their occurrence was due to the fact that for the two wet cells on the nested boundary adjacent to dry grid cells the quadratic interpolation method could not be used; instead the boundary values for these cells were linearly interpolated. On the whole, the results from the quadratic-interpolated nested model compare very favourably with those from the fine-grid model. Figures 11 and 12 compare velocities at low water (37.5 hrs) and mid-flood (40.625 hrs) for the fine-grid model and the quadratic-interpolated model. The results from the nested model can be seen to be almost identical to those predicted by the fine-grid model.

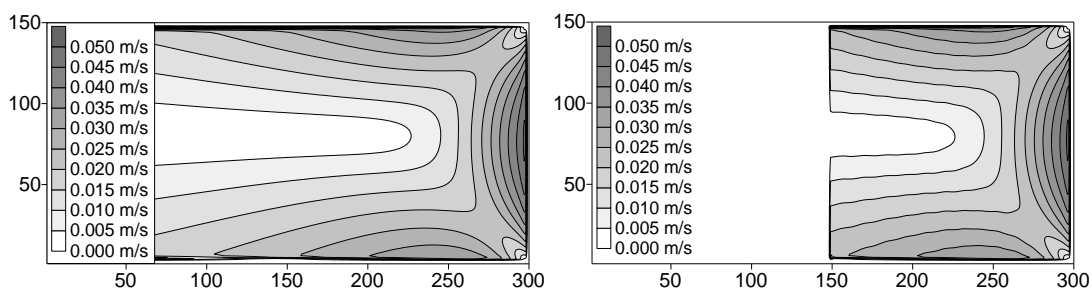


Figure 11: Comparison of current velocity between fine-grid model (left) and quadratic-interpolated nested model (right) at low water.

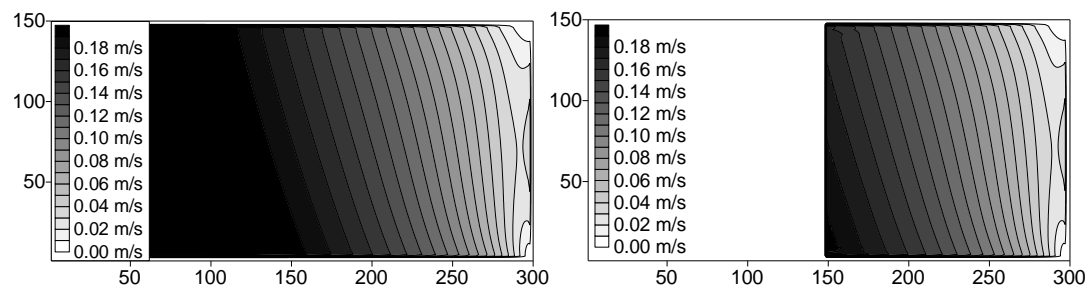


Figure 12: Comparison of current velocity between fine-grid model (left) and quadratic-interpolated nested model (right) at mid-flood.

## 5 Conclusions

A nested version of the hydrodynamic module of the two-dimensional DIVAST model has been successfully developed. The nested model predicts water elevations and current velocities within the nested domain to a similar degree of accuracy as a model using the same spatial resolution as the nested domain. It was shown that model results, in the form of water surface elevations and current velocities, vary when a higher spatial and temporal resolution is employed. This is to be expected. Variations in water surface elevation were insignificant, however, quite significant variations in current velocities occurred when the spatial and temporal resolution were improved from 120m to 40m and 120 to 60 seconds, respectively. It was also shown that the method of data transfer from the coarse-grid to the fine-grid open-boundaries is crucial to the accuracy of nested model computations. This was

demonstrated by comparisons of current velocities between the nested model and a fine-grid model.

The use of a zeroth-order spatial interpolation technique, whilst producing quite accurate results, caused perturbations in current velocity in the region of the nested open boundary. These perturbations were significantly reduced through the use of a linear spatial interpolation technique and were eliminated by the use of a quadratic-interpolation technique. Variations in water elevation between the fine-grid and nested models were insignificant at less than 0.4%. Variations in current velocity of up to 10% were recorded between the models, however, these variations only occurred when current velocities were very low and, accordingly, a small variation in magnitude represented a significant percentage error. Overall, the quadratic-interpolated nested model accurately computes the hydrodynamics in the nested domain.

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