Parameterization of turbulence models using 3DVAR data

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DYNAMIC PARAMETERIZATION OF TURBULENCE MODELS – A 3DVAR DATA ASSIMILATION APPROACH

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The research shows that the Prandtl mixing length model and the two-equation k-ε model, with default parameterization predefined according to literature recommendations, overestimate eddy viscosity which in turn results in a significant underestimation of velocity magnitudes in the harbour. The data assimilation of the model-predicted velocity and laboratory observations significantly improves model predictions for both turbulence models by adjusting modelled flows in the harbour to match de-errored observations. 3DVAR allows also to identify and quantify shortcomings of the numerical model. Such comprehensive analysis gives an optimal solution based on which numerical model parameters can be estimated. The process of turbulence model optimization by reparameterization and tuning towards optimal state led to new constants that may be potentially applied to complex turbulent flows, such as rapidly developing flows or recirculating flows

INTRODUCTION

Hydrodynamic modelling of coastal waterbodies facilitates improved scientific understanding of coastal processes such as horizontal and vertical circulation, mixing and dispersion, particularly with regard to their temporal and spatial variability. This knowledge is beneficial to a wide range of applications in the coastal and offshore engineering sector such as coastal structure design, marine renewable energy and pollutant transport. However, modelling marine hydrodynamics is a challenging task as it depends on the mathematical representation of the complex physical processes that govern the hydrodynamics.
The research presented in this paper extends the work recently published by Olbert et al. (2013). The main objective of this research is to optimize the performance of a hydrodynamic model by dynamically parameterizing turbulence schemes using data assimilation. The study aims to demonstrate the superiority of this novel approach to parameterization of turbulence models to the use of standard non-optimized models and statically parameterized models.
The research is described in three sections. First, outputs from the numerical simulations are compared with experimental observations from the tidal basin and shortcomings of the numerical model are identified and quantified. This is followed by details of the assimilation of laboratory data into the numerical model and the assessment of the performance of the 3DVAR

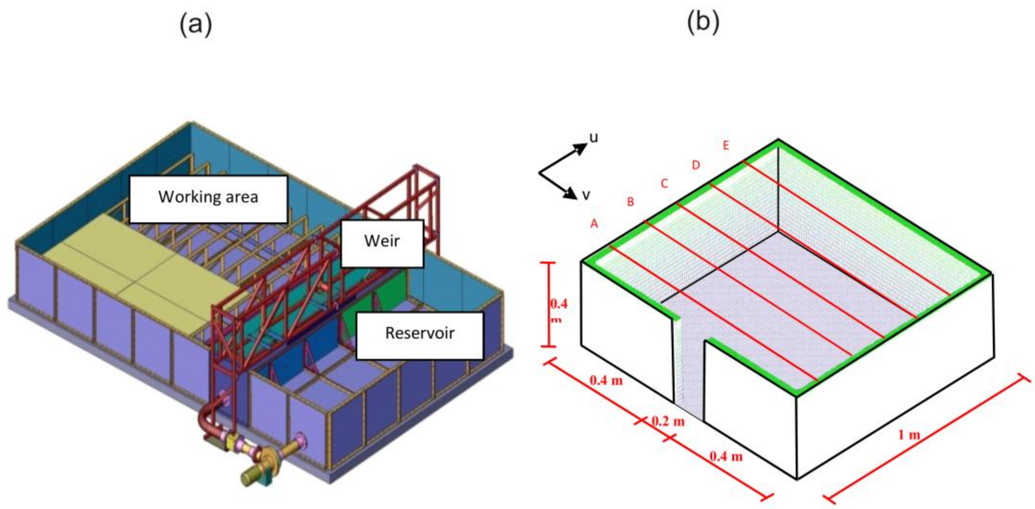


Figure 1. Schematic illustration of tidal basin (a) and harbour dimensions with locations of axes A-E (b).

assimilation technique. Finally, the turbulence schemes are statically and dynamically parameterized to improve hydrodynamic model performance by matching with an optimal solution obtained using data assimilation.

METHODS

Tidal basin

Assessments of numerical model performance and parameterization of turbulence schemes were based on comparisons with laboratory datasets of horizontal flows subject to data assimilation. The laboratory experiment was carried out in a tidal basin - a physical model designed to generate tides and tidally-induced water circulation. The basin is 8.0 m long by 5.0 wide tank with a total depth of 1.0 m. In horizontal plan it is divided into three sections: (1) the reservoir, (2) the manifold chamber and (3) the working area (see Fig. 1a). Water from the reservoir is pumped at a constant rate through a manifold into the manifold chamber, from where some portion of pumped water is moved further to the working area while the excess of water in manifold chamber is reversed to the storage reservoir via a weir overflow. The working area is separated from the manifold chamber by a honeycomb-shape porous baffle that promotes parallel flow into the working area and reduces swirl. Detailed description of the design and functionality of the tidal basin can be found in Olbert (2006).

Three-dimensional velocity components were recorded using a 10MHz Nortek Doppler Velocimeter (NDV) at a sampling rate of 25Hz. Flow measurements were sampled at 10 equidistant points along four axes A-D shown in Fig. 1b. At each of the 40 points sampled, data was recorded for 6-8 tidal cycles depending on repeatability of the velocity profile from one cycle to another. Then, velocity data were smoothed using a recursive filter and averaged over 5 tidal cycles to compute the mean velocity profiles for a tidal cycle.

Numerical model

The hydrodynamic modelling in this research was performed using DIVAST (Depth Integrated Velocity and Solute Transport), a two-dimensional, depth-integrated, finite difference model.

The hydrodynamic module is based on the continuity and Reynolds-averaged Navier-Stokes equations that include the effects of local and advective accelerations, the rotation of the earth, barotropic pressure gradients and bed resistance. The mathematical formulation of the DIVAST model can be found in Falconer and Liu (1995) while technical details and the parameterization is detailed in Falconer (1994). For brevity only a brief outline of the model is given here.

The model resolves the hydrodynamic equations using the hydrostatic assumption and the Boussinesq approximation. The finite difference method is adopted for spatial discretization in which a uniform space staggered Marker and Cell grid (Williams and Holmes, 1974) is applied horizontally on Cartesian coordinates. It calculates time-varying water surface elevations, velocity components and solute concentrations. The model is applicable in well-mixed coastal waters and estuaries where flow is predominantly horizontal and vertical accelerations are small in comparison to gravity effects.

Turbulence in DIVAST is assumed to be dominated by bottom friction; thus, eddy viscosity is calculated from the simple mixing length concept. In order to extend the model's applicability to flows where turbulence originates from horizontal shear-induced turbulence, the k-ε model was implemented. Bottom friction in DIVAST depends on the semi-empirical Chezy coefficient that, under turbulent flow, is assumed to be independent of the flow and to vary with the relative roughness of the bed.

The model domain represents the entire working area of the tidal basin and consists of 200x190 square grid cells of dimensions 0.025x0.025 m each. Tidal flows in the model are generated by a variable surface elevation prescribed at the open boundary as a radiation condition that relates the normal component of currents to the sea surface elevation accounting for tidal input. The numerical model setup and forcing represents exactly the same conditions as the physical model.

The selection of physical and numerical model geometry as well as tidal setup is not coincidental. The dimensions and flow regime in the harbour, although partly a result of tidal basin capacity, reflect real-world tidal conditions and harbour configurations when scaled up using the Froude scaling law. The 400:1 horizontal and 50:1 vertical length scales used for the physical model represent a large-size marina where flow regimes are induced primarily by semidiurnal tides of 12.4hrs period and 2.5 m amplitude that correspond to those typically observed along the Irish and British coasts.

In this research a two-equation scheme is tested to verify its applicability to complex hydrodynamic regimes.

3DVAR Data Assimilation and Parameterisation

The three-dimensional variational data assimilation (3DVAR) method of Courtier et al. (1998) was implemented in this research to optimize the performance of the numerical model with respect to various turbulence schemes. This method follows a multi-step algorithm that finds an optimal solution of the model by minimizing the cost function J(x). The cost function measures the distance between observations and the model as follows

$$J(x) = 1/2(x-x_b)^T B^{-1} (x-x_b) + 1/2 (H(x) - y)^T R^{-1} (H(x) - y) \qquad (1)$$

where x is the analysis velocity component, x_b is the background velocity component, y is the observational velocity component, B is the background error covariance matrix weighting the misfit between the analysis and background state, R is the observational error covariance matrix

weighing the misfit between the analysis state and the observation andH is the non-linear observational operator.
The performance of the data assimilation was assessed in parallel with computational cost of such system. In particular, the cost of data assimilation is often a downside of the system as it can be higher than the model cost depending on the algorithmic configuration. In this research, various algorithms for data interpolation and matrix inversion were tested prior to their application to 3DVAR. Ultimately, the bilinear interpolation algorithm of Kidner et al. (1999) was employed to map the first guess velocity components from model grid onto observation space. Two most common problems with matrix inversion such as computational errors due to imperfect computer arithmetic and the computational expense of the inversion were also explored in this research. In this regard, three methods, (1) Gauss, (2) LU and (3) Choleski decomposition, were examined. For a small dimension background error covariance matrix (10^4) as required in this research, inverse calculation was found to be efficient and computationally cheap for all three methods. The relative differences in the inverse products between all three methods were also found to be insignificant. The quality control method of Fuji et al. (2005) is adopted to improve analysis of the solution by judging the quality of data.

Static parameterization utilizes the optimization of initial conditions; the resulting tuned parameter is fixed in space and time for the duration of a simulation. The optimization of a hydrodynamic model using this method consists of 3 steps: (1) setting the initial turbulence parameters values, (2) running a simulation and (3) comparing the model output with the data assimilation solution. The readjustment of turbulence parameters is an iterative process and optimal turbulence parameter values are obtained when the modelled velocity is as close as possible to the assimilated velocity. Dynamic parameterization utilises turbulence parameters that make use of a functional relationship accounting for strong mean-strain flows. Turbulence parameter values are updated during numerical integration.

RESULTS AND DISCUSSION

In this section the predictive abilities of a hydrodynamic model based on data assimilation analysis, and on the static and dynamic parameterization of a turbulence scheme. Results are discussed separately.

Parameterization of *k*-ε model

The performance of the hydrodynamic model with the k-ε turbulence scheme employing default experimental coefficients is assessed in this section. The numerical model predicts jet-sink like circulation in the harbour interior in a similar manner to the physical model (Fig. 2). At mid flood, water entering the harbour is separated into two symmetrical counter rotating gyres; these structures occupy the interior of the harbour. While the size of these circulation cells is comparable to observations, the locations and shape differ. Similar to the PML model, the simulated gyres are more circular in shape with stagnation points closer to the harbour entrance relative to the observations, as shown on currents and vorticity diagrams in Fig. 2. The rate of spread of the jet entering the harbour is overpredicted by the model due to the strong momentum transfer to turbulence-driven secondary motions. As a result, advective longitudinal flow along the axis of the jet is reduced and decelerated; however, the rate of reduction is not as significant as in the case of the PML model. The overproduction of turbulence by the *k*-ε model is likely to result from an invalid assumption of approximate equilibrium between the

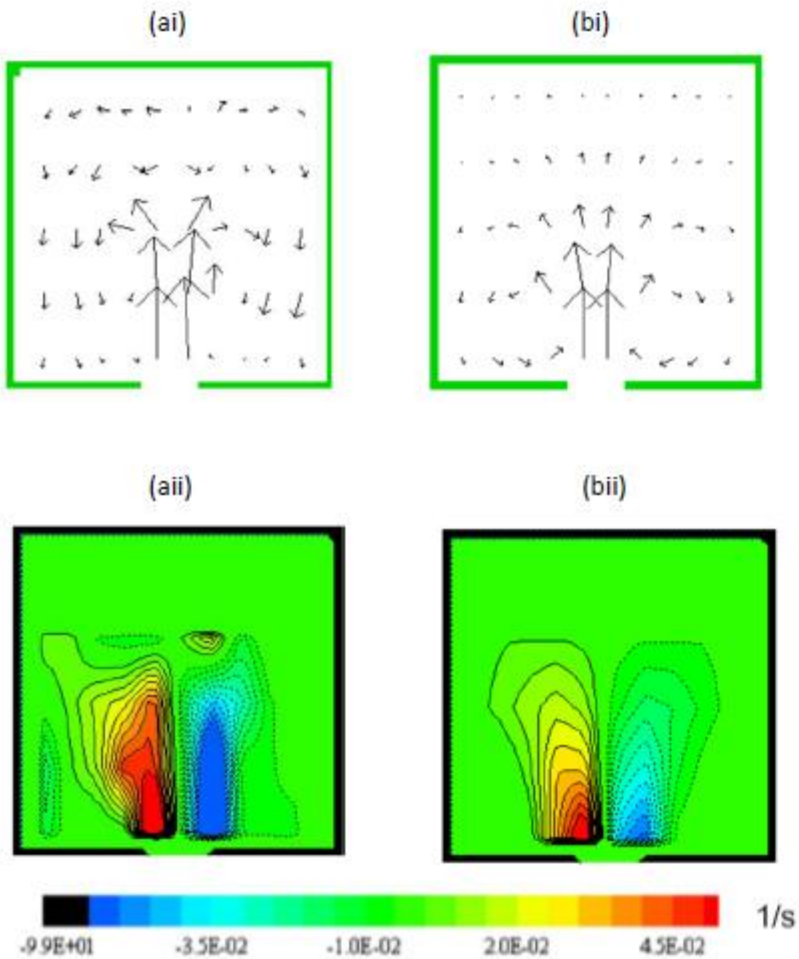


Figure 2. Flow pattern in the harbour: observations (a), and *k*-ε model (b), velocity (i) and vorticity (ii). Model simulated velocity and vorticity mapped on the observational grid for comparative purposes

production of kinetic energy of the turbulence motion and its dissipation. It becomes evident that in strong mean-strain flows such as the recirculating flows investigated in this research the production representing the transfer of kinetic energy from the mean to turbulent motion is not balanced by the dissipation. In order to account for increased production where mean-strain is high, the standard model should be modified in terms of some of its constants either through parameter tuning or functional relationships. The second approach in particular is likely to extend the universality and range of applicability of the model.

Dynamic parameterization of *k*-ε model

As shown in pervious sections the universality of constants in the *k*-ε model cannot be predetermined and for certain flows some of the constants may require different values. Olbert et al. (2013) presented a simple static parameterization approach where the difference between turbulence production and dissipation is accounted for through reparameterization of the production-dissipation parameter, C. This lengthy process is carried out via a computer optimization by fitting tuned model results to the assimilated optimal solution. The study

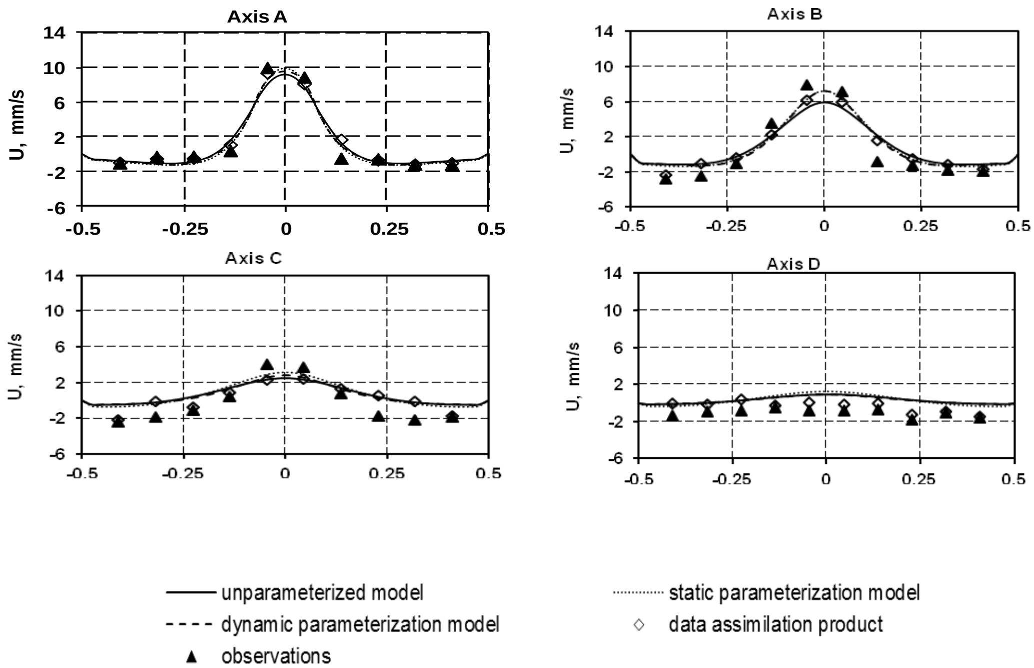
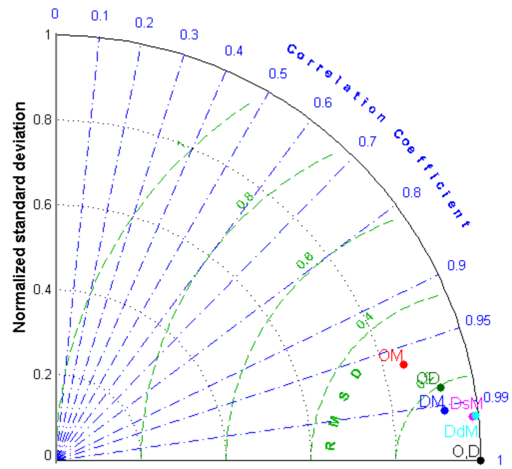


Figure 3. Observed, assimilated and non-assimilated velocity magnitudes along axes A-D in the harbour

showed that calculations were sensitive to the value of this parameter. The best model performance was obtained for C=0.085 yielding the production to dissipation ratio of approximately 1.2 as oppose to an equilibrium of 1.0. Velocities for the model with a fixed value retuned coefficient are shown in Fig. 3; the magnitudes are generally greater than those simulated by the unparameterized *k*-ε model and agree well with assimilated analysis magnitudes.

Similar to the PML model, the static parameterization of the *k*-ε model is a trial-and-error-based and labour-intensive process that depends on flow properties and therefore lacks complete universality. These limitations can be overcome by a dynamic approach where the constant is replaced with a variable that is a function of suitable flow parameters.

The dynamic approach used in this research makes use of an empirically-determined relationship between production of kinetic energy, P, and its dissipation, ε, developed by Rodi (1984). P is related to ε through an empirical function $C = f(P/\varepsilon)$ where C= 0.09 for $P = \varepsilon$, $C < 0.09$ for $P > \varepsilon$ and $C \sim 0.065$ for $P >> \varepsilon$. Here, C exhibits different values for different flow properties. As the model takes better account of changes in turbulence structure the predictive ability of the model improves. As shown in Fig. 3 velocity magnitudes obtained using dynamic parameterization show good agreement with the data assimilation optimal solution. Along axis A the normalized standard deviation and correlation coefficient are close to unity the RMSD is close to zero as would be anticipated for a very good agreement (Fig. 4). These encouraging results imply that the dynamic parameterization accurately optimizes turbulence processes. The empirical relationship used in this method extends the applicability of the standard model to flows with non-equilibrium production-dissipation characteristics. Additionally, the calculations require no extra computational cost. The dynamic parameterization is a self-updating process



● O – observation ● D – DA ● DM – DA vs unparameterized

● OM – observation vs unparameterized ● DsM – DA vs static parameterization

● OD – observation vs DA solution ● DdM – DA vs dynamic parameterization

Figure 4. Taylors diagram of U-velocity component

that provides a generic solution without the necessity for labour-intensive and time consuming multistep model tuning as required in static approach.

CONCLUSIONS

In this research the viability of static and dynamic parameterization of turbulence models is explored. The study extends the work of Olbert et al. (2013) where simple parameterization (termed here static parameterization herein) was preliminary examined; a dynamic parameterization is a novel aspect of this research. Two turbulence closure schemes the Prandtl mixing length model and the two-equation *k*-ε model were incorporated into a numerical model and examined with respect to their universality of application, complexity of solutions, computational efficiency and numerical stability. Current velocities measured in a physical model were assimilated into a hydrodynamic model using a 3DVAR data assimilation scheme in order to improve prediction skill of the model in regions where turbulent processes are of importance. A square harbour with one symmetrical entrance subject to tide-induced flows was selected to investigate the structure of turbulent flows and the model's ability to reproduce those structures. The experimental part of the research was conducted in a tidal basin. A significant advantage of such laboratory experiments is that it is a fully controlled environment where domain setup and forcing are well defined.

The main findings from this research regarding parameterization of turbulence schemes are summarised here:

(i) The research demonstrates that 3DVAR can be utilized to identify and quantify shortcomings of the numerical model and lead to an improvement of model forecasting by a correct parameterization of the turbulence schemes.

(ii) The data assimilation significantly enhances model predictions for both turbulence models. Although there are discrepancies in the flow magnitudes between the two turbulence models (particularly in the inflow region) the assimilated products are in close agreement. The analysis product becomes an optimal solution towards which the turbulence models should be tuned.

(iii) The output of the data assimilation and verification results suggest that fundamental problems of the 3DVAR method such as (a) identification of the model and observation errors, (b) parameterization of spatial correlations and (c) specification of the background error covariance matrix, which quality of analysis is most sensitive to, are correctly resolved in the study.

The research presented in this paper demonstrates that the application of model parameterization in conjunction with data assimilation is a very useful tool that may greatly benefit both oceanographers and coastal engineering communities. On the one hand, the method allows improved understanding of coastal dynamics and monitoring of coastal systems without the necessity of implementation of expensive coastal observation systems. On the other hand, model parameterization through data assimilation can be successfully used in a variety of coastal engineering applications where accuracy of hydrodynamic predictions is central to a design and built of costal structures.

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