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Modulus of elasticity prediction through transversal vibration in cantilever beams and ultrasound technique of different wood species

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ABSTRACT

The prediction of the modulus of elasticity (MOE) of five species of different spectrum density woods, namely, *Populus* × *euramericana* I-214 (Poplar), *Fagus sylvatica* L. (Beech), *Quercus pyrenaica* L. (Oak), *Paulownia elongata* S.Y.Hu (Paulownia) and *Pinus sylvestris* L. (Scots pine) were examined through the natural frequency of vibration on cantilevered beams (transverse direction) and ultrasound (longitudinal direction). Cantilever beams are commonly used for other materials but limited information is available for wood materials tested in this manner. A total of 60 specimens with nominal dimensions of 40 × 60 × 1200 mm^3 were tested, which were visually graded according to UNE 56544:2022 and UNE 56546:2022 as first class, and finally the global bending stiffness was obtained from a four-point bending test. Utilising this data, a regression model was presented to predict the MOE.

Also, *Picea sitchensis* Trautv. & G.Mey (Sitka spruce) has been chosen as a blind species in order to validate the regression model of prediction of the MOE as a function of the dynamic MOE by ultrasound. Bending strength, modulus of elasticity and density were obtained according to the EN 408. In the prediction model using the dynamic MOE with vibrations, an r^2 of 95.9% was achieved for the induced vibration technique which was found to be slightly higher than the model for the ultrasound prediction which had an r^2 of 93.7%.

1. Introduction

In general, non-destructive testing techniques (NDT) applied to wood or wood-derived components are tools that can estimate the physical–mechanical properties and determine and ensure their structural integrity during construction. Furthermore, such techniques can identify damaged or deteriorated components without compromising the functionality of the structural element. To this end, a great number of studies have been performed in the last few decades using different methods such as stress wave techniques, transverse vibration, ultrasounds, X-ray or thermography, among other techniques [1–28].

One NDT that has gained importance in the last few years is the induced vibration technique which measures acoustic properties to predict and evaluate the physical–mechanical properties of wood and establish classification ranges based on these properties [7,14,29]. In this regard, studies have focused on estimating the behaviour of wood or wood-based components based on vibration frequencies (longitudinal and transverse), estimating their dynamic elastic modulus, and analysing the relationships between the dynamic parameters and the static variables of elasticity and resistance, namely the static modulus of elasticity and bending strength [30–32]. Similarly, several research projects have focused on studying the relationship between dynamic parameters and physical variables and between dynamic parameters and quality parameters, such as the presence of defects in the wood. In most of these studies, vibration analyses have been performed on simply supported beams and beams with both ends free [33,34]. Their conclusions are all similar: dynamic properties correlate sufficiently well with static variables, namely the modulus of elasticity and to a lesser degree with the bending strength, which can suitably predict the physical characteristics and quality structural parameters of wood [7,16,41,20,29,35–40].

On the other hand, ultrasonic techniques are one of the most widely

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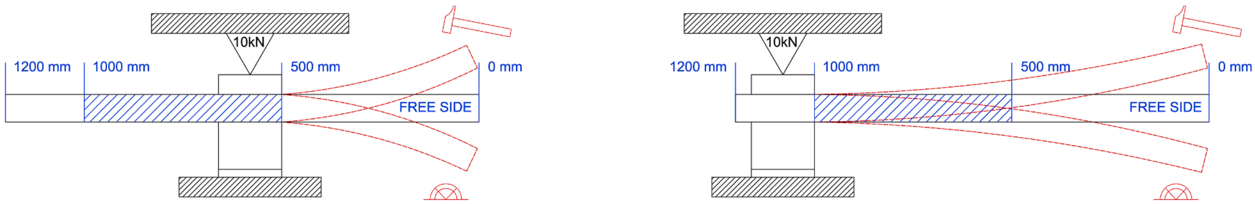

Fig. 1. Configuration of the cantilever beam. a) initial length, b) final length.

used non-destructive testing methods for timber, due to their sufficient accuracy and relative simplicity of testing. Thus, many studies have been carried out for timber grading and for non-destructive evaluation of in-use timber structures [42–47].

Although in order to grade determining timber properties by different methods that use this research approach (induced vibrational) [48], there is essentially no study or grading machine development based on the application of cantilever beams to evaluate the behaviour of wooden beams. However, there are a large number of projects that have utilised this approach but they are focused on different isotropic materials [16,49–58]. By definition, a cantilever has a rigid support on one end, which prevents any type of displacement or rotation, and the other end is free to vibrate [58]. Therefore, the behaviour of wood, which is an anisotropic and heterogeneous material when fixed in a cantilever may be an appropriate alternative to determine its vibrational behaviour and estimate its elastic behaviour. Furthermore, this type of support is easily reproducible in the laboratory.

The aim of this study was to determine the elastic behaviour of different timber species (with low, medium and high density) by determining the dynamic modulus of elasticity based on transversal vibration frequencies in cantilever beams, and compare them with ultrasonic and static bending tests.

2. Materials and methods

2.1. Testing materials

The testing was conducted using five different wood species, some of them commonly used in construction and structures such as *Populus* × *euramericana* I-214 (Poplar), *Fagus sylvatica* L. (Beech), *Quercus pyrenaica* L. (Oak), *Paulownia elongate S.Y. Hu* (Paulownia) and *Pinus sylvestris* L. (Scots pine). The test material consisted of 60 wooden joists (12 pieces per species) of 40 mm × 60 mm and 1200 mm (structural proportion dimensions, EN 408:2011 [59]) that were visual graded according to UNE 56544:2022 [60] and UNE 56546:2022 [61] and achieved a first class. The test material was conditioned in Wood Laboratory of Valladolid University (Spain) at 65 ± 5 % HR and 20 ± 2 °C until reaching constant mass. All pieces were measured and weighed to calculate their density at 12 % moisture content (*MC*), according to standard EN 384:2016 [62].

In order to validate and ensure that the statistical model worked properly, a blind control sample comprising 10 *Picea sitchensis* Trautv. & G.Mey (Sitka spruce), of Irish provenance, with dimensions of 40 mm × 100 mm × 4500 mm in length and 12 % MC, was randomly selected (from a batch > 200 units) and was tested. This additional wood species was chosen because it has a wide range of elasticity values and is widely used in the European market. These tests were realised at the Timber Engineering Laboratory of the College of Science and Engineering at the University of Galway in Ireland.

2.2. Non-destructive testing by transversal free vibration

Each piece was placed in a cantilever configuration (one end fixed and the other end free) and clamped under a load of 10 kN to provide the optimum rigidity conditions. The test beams were attached to the main support using wood components and clamps to ensure uniform pressure. The initial free span length was 500 mm, which was later increased by a length, *d,* of 25 mm for each new position until a free span length of 1000 mm was achieved (Fig. 1).

The tests were carried out using 21 different free span lengths (500 mm/25 mm each step) for five species (Paulownia, Poplar, Scotland Pine, Beech and Oak) and, for each one, four tests were conducted for the two orientations of the section: vertical, *V*, and horizontal, *H*, where a total of 1008 tests per species and orientation (21 span lengths/orientation × 4 repetitions/span length × 12 beams/species) were completed. Seven free span lengths were tested for Spruce, where a total of 280 tests per orientation (7 span lengths/orientation × 4 repetitions/span length × 10 beams) were completed.

The vibration frequency in each cantilever beam was generated by applying a hammer blow perpendicularly onto the top part of the transverse face of the piece followed by measuring and recording the vibration using a signal receiver (microphone) and wave analyser (FFT analyser).

The natural frequency of transversal vibrations of a cantilever beam, according to the Euler-Bernoulli theory [63], can be calculated by Equation (1):

$$f_n = C_n\sqrt{\frac{(E_{dyn}*I)}{\rho*S*L^4}} \tag{1}$$

where f_n is the natural frequency (Hz), C_n is a constant that depends on the vibration mode (n), e.g. n_1 = fundamental mode, n_2 = first harmonic, etc., taking the values: mode 1 = 0.5602, mode 2 = 5.5014, mode 3 = 9.8198…; E_{dyn} is the dynamic modulus of elasticity (Pa); I is the area moment of inertia of the beam cross-section (m^4); ρ is the bulk density (kg/m^3); S is the area of the cross-section (m^2) and L is the free span length of the cantilever beam (m).

Equation (1) can also be rewritten as per Equation (2):

$$f_n^{\,2} = C_n^{\,2}*\frac{(E_{dyn}*I)}{\rho*S}*\frac{1}{L^4} \tag{2}$$

Equation (2) is similar to the general equation for a linear model, $y = mx + b$, in the form slope (m) –intercept (b) if it is assumed $y = f_n^2$ and $\times = (1/L^4)$, $m = C_n^2*(E_{dyn}*I)*(\rho*S)^{-1}$ and $b = 0$. In this way, for each free length (L), it will be possible to obtain a frequency value. Thus, plotting (x,y) points, the slope can be determined and the dynamic modulus of elasticity will be calculated as is indicated in Equation (3):

$$E_{dyn} = m*\frac{\rho*S}{C_n^{\,2}*I} \tag{3}$$

where m is the slope of the regression line ($x = 1/L^4$, $y = f^2$).

2.3. Non-destructive testing by ultrasonic technique

In addition to the NDT by transversal free vibration, all pieces were tested by the Sylvatest DUO ultrasound equipment (CBS-CBT, Lausanne, Switzerland) with conical 22 kHz sensors. The transducers were placed at the ends of the samples (direct method).

The dynamic modulus of elasticity ($E_{din\ ultra}$, MPa) was obtained according to Equation (4).

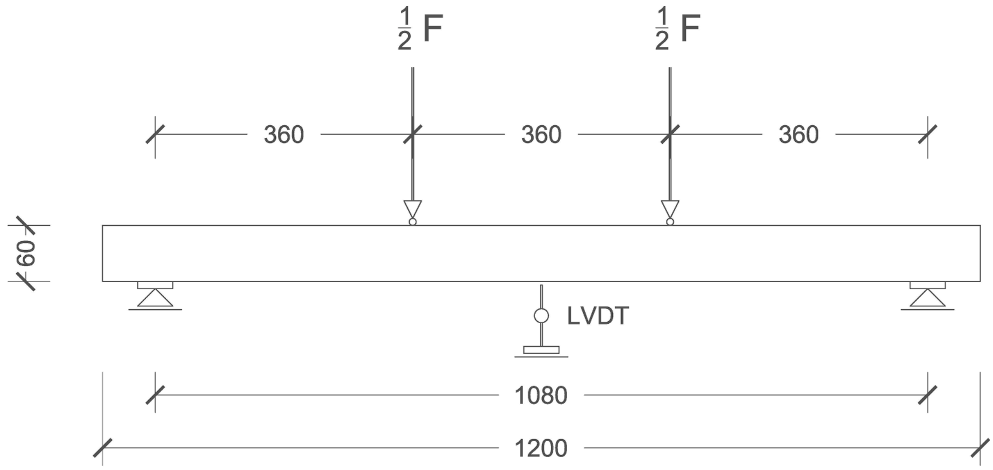

Fig. 2. Static bending testing according to EN 408:2011 [59] (dimensions in mm).

$$E_{din\ ultra} = V^2 \cdot \rho \tag{4}$$

Where V is the ultrasonic velocity (m/s) and ρ is the density (kg/m^3) of each piece.

2.4. Density and static bending testing

Finally, every wood sample was tested in a universal testing machine following a four-point bending test configuration in accordance with standard EN 408: 2011 [59]. The test configuration is shown in Fig. 2.

The global bending modulus of elasticity (MOEGTO) was obtained by recording the deformations measured during the test with a linear variable differential transformer (LVDT) located at the center of the span. The tests were performed with a constant displacement speed of 10 mm/min. The section of the stress–strain curve for which the beam's stiffness was calculated corresponded to between 10 % and 40 % of the estimated maximum load for all beams. When 40 % of the estimated maximum load was reached the test was finished and the sample was removed without any damage. The MOEGTO was determined according to Equation (5) (EN 408:2011 [59]).

$$MOEGTO = \frac{3al^2 - 4a^3}{2bh^3\left(2\frac{w2-w1}{F2-F1} - \frac{6a}{5Gbh}\right)} \tag{5}$$

Where a (mm) is the distance between a loading point and the nearest support in a bending test, l (mm) is the distance between supports, b (mm) is the width of the cross-section, h (mm) is the height of cross-section, w (mm) is the deformation, F (N) is the force, G (MPa) is the transverse modulus of elasticity. In this case, G was taken as infinite according to EN 408:2011 [59].

2.5. Statistical treatment of experimental data

All statistical analyses were performed using R software (v. 4.1.1:2021) [64]. The assumptions of independence, normality, and homoscedasticity were checked for the variables under study ($E_{din\ vibra}$, $E_{din\ ultra}$, MOEGTO, and density). The normality of the data was tested for all populations using the Shapiro–Wilk normality test and Q–Q normal probability plots. The homoscedasticity requirement was contrasted by Bartlett's test. Since it was not met on numerous occasions, the usual comparative analysis by ANOVA could not be used. Robust comparison methods Welch's heteroscedastic F-test with trimmed means and Winsorized variances were used instead. This robust procedure tests for equality of means by replacing the usual means and variances with trimmed means and Winsorized variances [65,66], together with bootstrapping and comparison using robust homogeneous groups.

For the validation of the models, a 10-fold cross-validation method was used. 10-fold cross validation is a method used to evaluate the accuracy and performance of a machine learning model. The objective is to determine the reliability of a model's predictions when it is trained and tested on different data sets. In this method, the data set is divided into 10 equal parts or folds. 9 of the folds are used to train the model while the remaining fold is used to test it. This process is repeated 10 times, with each fold serving as the test set once. The 10-fold cross validation helps to reduce the risk of overfitting, which is when a model is too closely tailored to the training data and does not perform well on new data. By testing the model on different sets of data, the 10-fold cross validation provides a better estimate of its performance on new data. The results from the 10 tests are then averaged to obtain a final score, which provides a more accurate representation of the model's performance. The objective of 10-fold cross validation is to create a more

Table 1
Descriptive analyses: density.

Specie	Density (kg/m^3)			Normality test		Homoscedasticity test	Robust homog. Groups
				Shapiro-Wilk		Bartlett test	
	Mean ± IC**	CV (%)	range	W	p-value	p-value	
Paulownia elongata	243.35 ± 14.23	9.2	68.0	0.913	0.234	0.026 *	e
Populus × euramericana	389.37 ± 28.56	11.5	151.5	0.974	0.954		d
Picea sitchensis	409.97 ± 27.37	9.3	109.6	0.903	0.2339		d
Pinus sylvestris	569.43 ± 19.10	5.3	94.2	0.945	0.574		c
Fagus sylvatica	672.75 ± 36.24	8.5	133.0	0.793	0.008 *		b
Quercus pyrenaica	766.16 ± 37.90	7.8	194.0	0.864	0.055		a

* p-value < 0.05 implies that the assumption of homoscedasticity is not met.
** IC 95% Interval of Confidence.

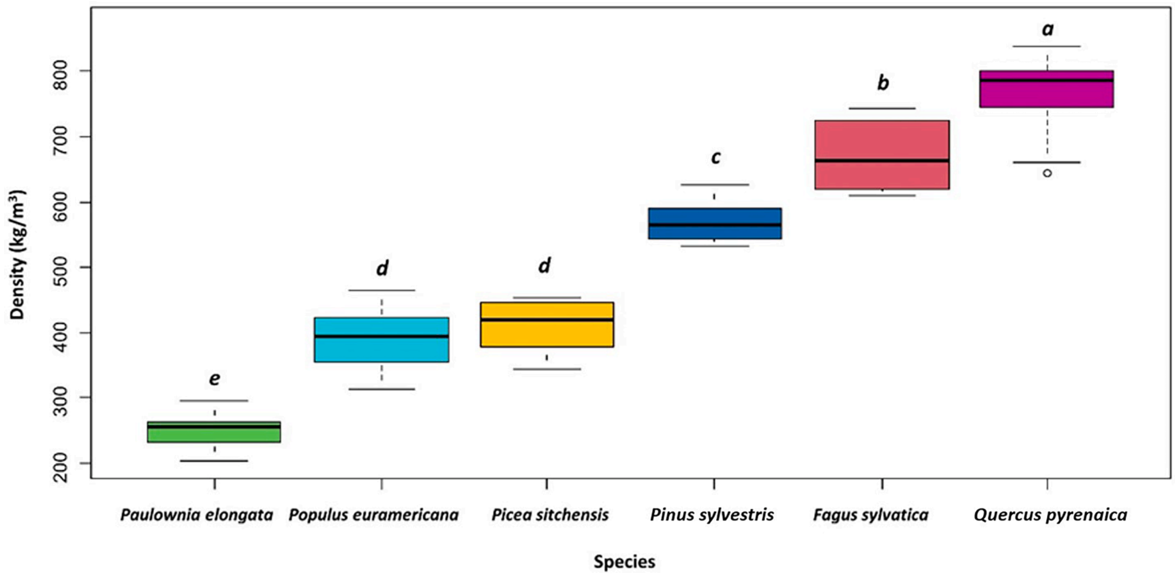

Fig. 3. Density box-plots of each species tested.

Table 2
Descriptive analyses: MOEGTO.

Specie	MOEGTO (MPa)			Normality test		Homoscedasticity test	Robust homog. Groups
				Shapiro-Wilk		Bartlett test	
	Mean ± IC**	CV (%)	range	W	p-value	p-value	
Paulownia elongata	4933.1 ± 186.9	6.0	849.1	0.925	0.335	2.87e-04 *	d
Populus × *euramericana*	9173.9 ± 1009.3	17.3	5081.3	0.959	0.767		c
Picea sitchensis	6649.3 ± 1237.1	26.0	4620.3	0.904	0.242		c d
Pinus sylvestris	13462.6 ± 365.4	4.3	2012	0.978	0.976		a
Fagus sylvatica	12139.0 ± 542.2	7.0	2429	0.844	0.031		b
Quercus pyrenaica	12437.3 ± 934.3	11.8	4908.7	0.945	0.568		a b

* p-value < 0.05 implies that the assumption of homoscedasticity is not met.
** IC 95% Interval of Confidence.

reliable and robust model by validating its accuracy on multiple data sets [67]. In this case, each cross-validated (10 fold) was repeated 5 times and the average statistical values were obtained. To obtain more robust estimators, the operation was repeated 200 times and the mean values were obtained. Finally, another model check was performed, involving inputting values from a spruce species with distinct geometric and anatomical characteristics from those used to create the model (as was described in 2.1. Testing materials) and verifying that they fall

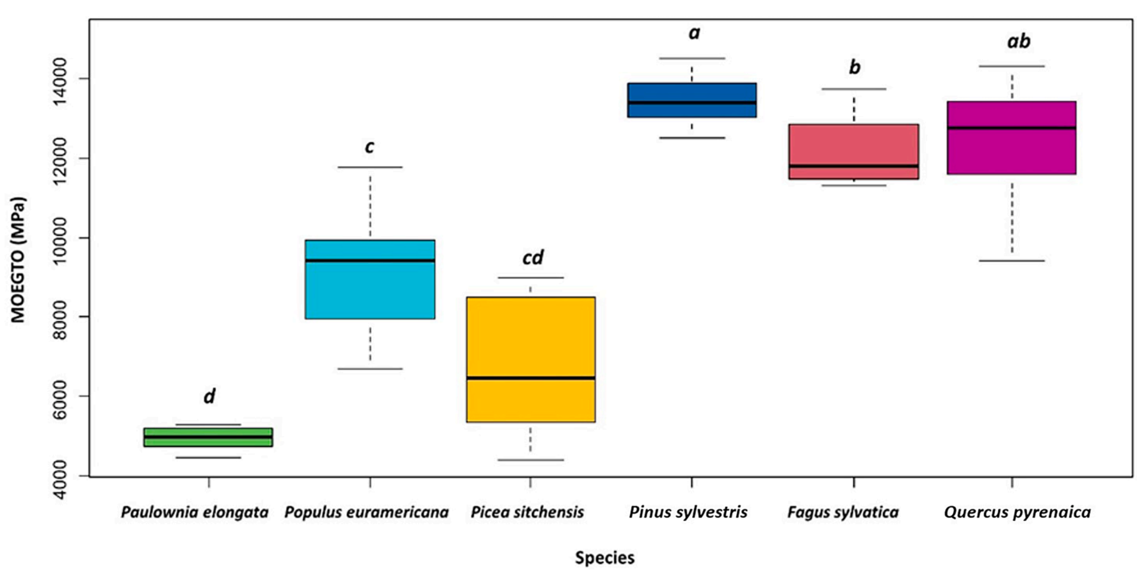

Fig. 4. MOEGTO box plot of each species tested.

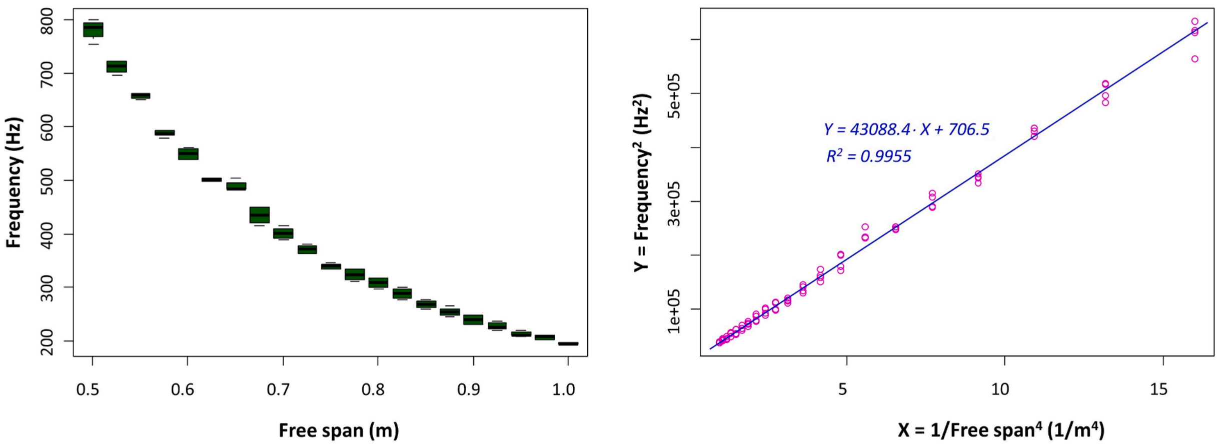

Fig. 5. Paulownia's example of the relationship between cantilever beam free span to vibration and its first natural frequency of vibration (left), and its transformation to linearise this relationship (right).

Table 3
Regression lineal ($x = 1/L^4$, $y = f^2$) Residual Standard Error (RSE) and multiple R-squared per specie.

Specie	Vertical (V)				Horizontal (H)			
	RSE		Multiple R-squared		RSE		Multiple R-squared	
	Min	Max	Min	Max	Min	Max	Min	Max
Paulownia elongata	15,750	31,380	0.993	0.998	7519	12,090	0.994	0.997
Populus × euramericana	44,230	65,960	0.963	0.984	22,940	37,210	0.951	0.976
Pinus sylvestris	36,860	72,190	0.957	0.983	2981	11,490	0.995	0.999
Fagus sylvatica	148,700	418,500	0.974	0.998	60,120	88,020	0.993	0.998
Quercus pyrenaica	124,800	216,800	0.995	0.998	68,930	110,200	0.993	0.997

within the model's prediction range. This allows for assessing the accuracy of the model's performance in the other five species.

3. Results and discussion

3.1. Density and static bending testing

Descriptive analyses of the density values are shown in Table 1:

Where it can be seen that the mean value of density per species ranged between 243 kg/m^3 for Paulownia and 766 kg/m^3 for Oak. The robust homogeneous group analysis establishes that there are statistically significant differences between all species except Poplar and Spruce (Fig. 3).

The density values were similar to those published for these species by other authors in *Populus × euramericana* I-214 [8,12,68] in *Pinus sylvestris*, [30] in *Pinus nigra*, [33] in *Quercus pyrenaica* and *Fagus sylvatica*, [69] in *Paulownia elongata*, and [34] in *Picea sitchensis*.

Descriptive analyses of MOEGTO values are shown in Table 2.

It can be seen that the different species cover a wide range of mean values from approximately 4900 MPa for Paulownia to 13500 MPa for Scots pine. The high value of Scots pine, which forms a homogeneous group with oak, is noteworthy. It is also worth noting Spruce's low value and high variability, mainly because the pieces present a greater number of singularities as they come from an unclassified batch (Fig. 4).

The MOEGTO values in *Populus × euramericana, Pinus sylvestris, Quercus pyrenaica* and *Fagus sylvatica* were higher than those published for the same species by other authors [8,12] in poplar, [30,68], in pinus, and [33] in oak. This is due to the fact that the pieces tested were free of singularities or defects that affect the mechanical properties of the wood. However, in the case of *Picea sitchensis*, the values were lower than those published by other authors [34]. This could be because they were pieces with knots and juvenile wood in which the bending properties are lower [70].

3.2. Non-destructive testing by transversal free vibration

First, $E_{din\ vibra}$ slope (*m*) was calculated using the values obtained for each of the 4 trials per free span. Fig. 5 shows an example of the Paulownia relationship between free beam length to vibration and its first natural frequency of vibration, and its transformation to linearise this relationship.

The results obtained for the $E_{din\ vibra}$ slope (*m*) of all the species tested in the two directions are shown in Table 3.

It can be seen that the R^2 values in all cases are higher than 95 % ($R^2 \geq 0.95$). This makes it possible to determine the value of the slope with great accuracy. The values for spruce are not reflected as they are only

Table 4
Transversal vibration dynamic modulus of elasticity $E_{din\ vibra}$ (*H* and *V*) per species.

Specie	$E_{din\ vibra}$ *V* (MPa)		$E_{din\ vibra}$ *H* (MPa)		Paired *t*-test
	Mean* (CV%)	Min – Max* (p value SW test)	Mean* (CV%)	Min – Max* (p value SW test)	$E_{din}V$ vs $E_{din}H$ p-value
Paulownia elongata	5239.8 (12.0)	4251.1 – 6636.6 (0.791)	5143.5 (11.3)	4280.7 – 6497.6 (0.618)	0.0690
Populus × euramericana	9271.2 (16.54)	6996.2 – 11860.5 (0.636)	9354.9 (16.9)	6733.0–11691.2 (0.356)	0.3211
Pinus sylvestris	13105.4 (5.0)	11692.7 – 14047.1 (0.648)	13155.14 (4.3)	12140.0 – 13804.7 (0.262)	0.5603
Fagus sylvatica	13855.4 (8.6)	12155.2 – 15412.4 (0.730)	13918.5 (7.4)	12205.3 – 15353.8 (0.562)	0.2881
Quercus pyrenaica	14493.5 (8.6)	12178.6 – 16038.0 (0.131)	14566.8 (10.1)	11987.1 – 16863.4 (0.899)	0.9957

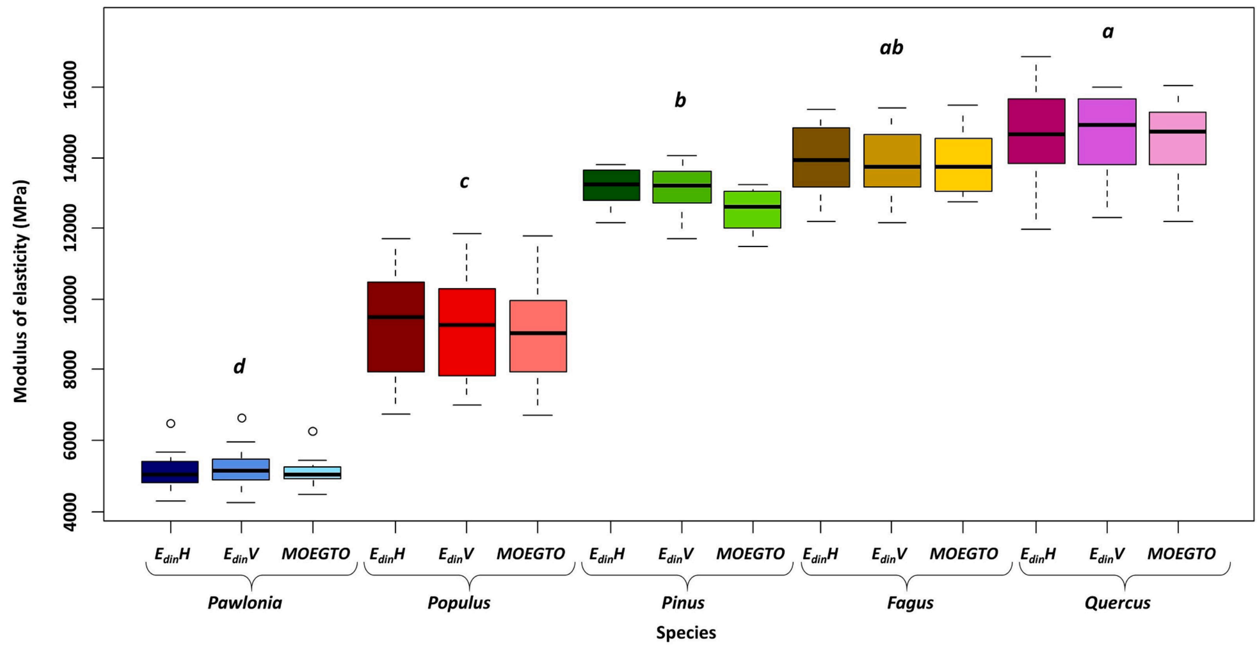

Fig. 6. Box plot of horizontal $E_{din\ vibra}$, vertical $E_{din\ vibra}$, and MOEGTO per species.

Table 5
Ultrasound dynamic modulus of elasticity.

Specie	$E_{din\ ultra}$ (MPa)			Normality test Shapiro-Wilk		Homoscedasticity test Barlett test	Robust homog. Groups
	Mean ± IC	CV	range	W	p-value	p-value	
Paulownia elongata	7060.8 ± 1009.0	22.5	68.0	0.903	0.176	0.561	d
Populus × euramericana	12570.5 ± 1304.9	16.3	6598.1	0.926	0.335		b
Picea sitchensis	9405.0 ± 1539.0	22.9	14617.5	0.976	0.937		c
Pinus sylvestris	16780.3 ± 652.3	6.1	3198.9	0.930	0.377		a
Fagus sylvatica	15894.6 ± 937.4	9.3	4465.9	0.942	0.528		a
Quercus pyrenaica	15917.1 ± 1133.9	11.2	194.0	0.964	0.841		a

tested in one orientation as a model check.

Table 4 shows the mean values, dispersion values, and normality tests for the transverse vibration dynamic modulus of elasticity ($E_{din\ vibra}$) for vertical (V) and horizontal (H) orientation.

There were no statistically significant differences between the values obtained in the horizontal or vertical direction (p-value $>$ 0.05) as shown by the pairwise comparisons for each species, which coincides with other authors [7,68,71] in different pine species.

The mean E_{din} values were higher than those published by other authors in simply supported vibration tests because the pieces analysed in this study were free of defects compared to the lower values in structural wood of the same species [7,8,12,33,68].

The comparisons made between the values of horizontal $E_{din\ vibra}$, vertical $E_{din\ vibra}$, and MOEGTO by species were similar in all cases. Homogeneous groups identified alphabetically were established as shown in Fig. 6.

3.3. Non-destructive testing by ultrasonic technique

Table 5 shows the mean values, dispersion values, and normality tests for the ultrasound time of flight dynamic MOE ($E_{din\ ultra}$). A robust homogeneous groups test is also presented for each species.

The mean $E_{din,ultra}$ values in *Populus × euramericana*, *Pinus sylvestris*, *Quercus pyrenaica* and *Fagus sylvatica* were higher than those published for the same species by other authors [8,12] in poplar, [7,30,68] in pinus, [33] in oak. This is likely due to the fact that the pieces tested

Table 6
MOEGTO prediction based on dynamic modulus of elasticity.

Species	MOEGTO vs $E_{din\ ultra}$			MOEGTO vs $E_{din\ vibra}$		
	p-value	RSE	Adjusted R^2	(p-value)	RSE	Adjusted R^2
Paulownia elongata	1.710 e-03	184.4	0.607	(4.0 e-04)	160.3	0.703
Populus × euramericana	5.09 e-04	886	0.689	(2.24 e-04)	818	0.735
*Picea sitchensis**	8.1 e-04	919.5	0.714	(1.4 e-03)	903.8	0.719
Pinus sylvestris	7.29 e-03	413.7	0.582	(2.08 e-05)	234.5	0.836
Fagus sylvatica	9.41 e-04	505	0.649	(9.7 e-05)	404.9	0.775
Quercus pyrenaica	2.22 e-04	756.5	0.735	(1.2 e-04)	712.7	0.765
Global	2.2e-16	851.7	0.937	(2.2e-16)	685.3	0.959

*not included in the global model calculation.

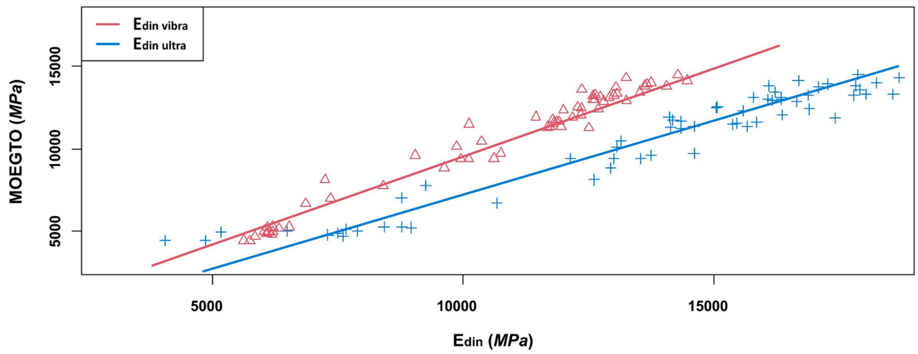

Fig. 7. Regression models $E_{din\ vibra}$ vs $E_{din\ ultra}$.

Table 7
Robust cross-validation tests indicators mean: RMSE, R^2, and MAE.

	Vibration	SD	Ultrasound	SD
Intercept (σ_{Int})	−1559.266	56.158	−766.2172	171.732
Slope (σ_{Slope})	1.125	$6.26\cdot 10^{-3}$	0.816	$1.20\cdot 10^{-2}$
RMSE (σ_{RMSE})	664.528	297.519	853.939	182.247
R^2 (σ^2_R)	0.972	$2.51\cdot 10^{-2}$	0.958	$2.38\cdot 10^{-2}$
MAE(σ_{MAE})	525.8918	198.680	756.524	172.104

were free of singularities or defects that affect the mechanical properties of the wood.

3.4. MOEGTO prediction based on dynamic modulus of elasticity.

Table 6 shows the values of the main statistics of the regression models of the predictor variables ($E_{din\ vibra}$ and $E_{din\ ultra}$) concerning MOEGTO.

It can be seen that, for each species, the adjusted R^2 values were higher for vibration than for the ultrasound technique. The Residual Standard Error (RSE) value shows that, in all cases, the errors of the vibration model were lower than in the case of the ultrasound model.

The mean values of the dynamic modulus of elasticity obtained by ultrasound were higher than those obtained with the vibration method, as is shown in Fig. 7, an aspect that has already been reflected in previous works comparing both methods of non-destructive classification [7,12,33,68,72–74].

3.5. Models validation

The results of the robust cross-validation tests are shown in Table 7. The smaller the root mean square error (RMSE) and mean absolute

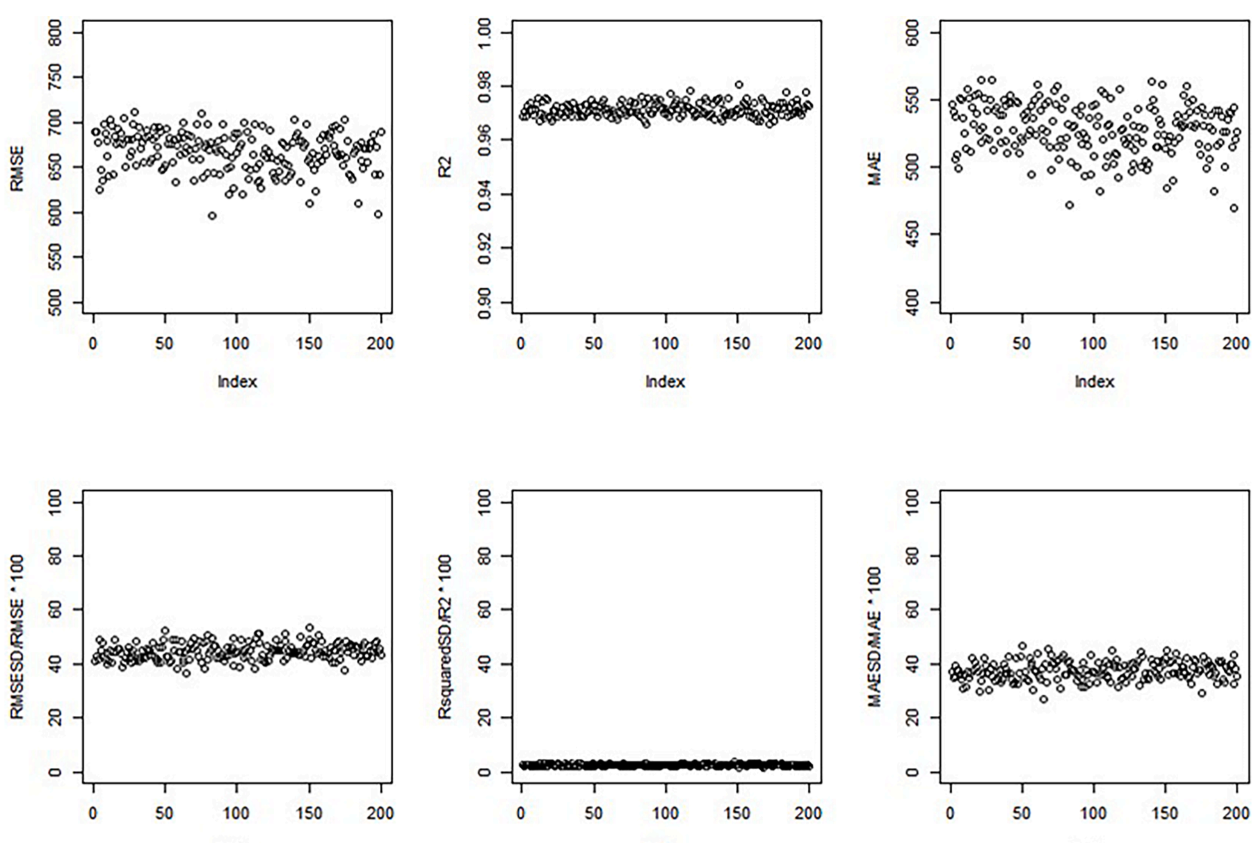

Fig. 8. Plot of the statistics for each of the 200 iterations of the cross-validation tests.

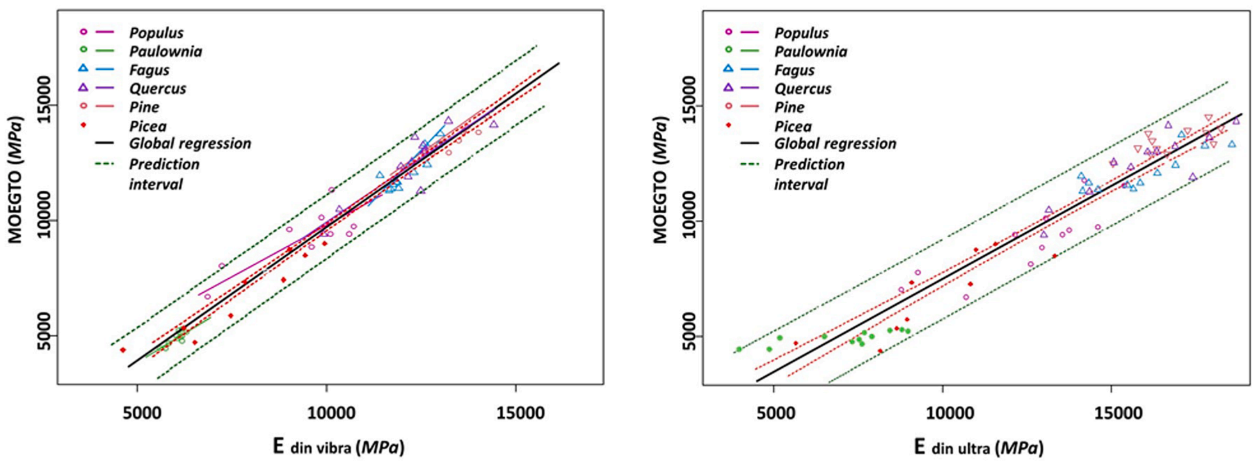

Fig. 9. General models MOEGTO vs $E_{din\ vibra}$, and MOEGTO vs $E_{din\ ultra}$.

error (MAE) value the better, and while the larger the R^2, too better. Consequently, the MOEGTO estimation model using transversal vibrations was the best one. Fig. 8 shows the values of the statistics for the 200 iterations of the cross-validation test. The robustness of the model is shown by the uniformity of the values of the statistics.

This is also observed in Fig. 9, where the dotted lines are further apart, i.e. to get the prediction right it needs more "width" concerning the regression line compared to vibrations, which in all statistics is better. These results coincide with other published articles in different pine species [7,68,71]. It can be seen that the prediction limits of the global regression contain all the points of the new species (*Picea sitchensis*).

4. Conclusions

The behaviour of the fundamental vibration frequency obtained for wood samples (*Populus* × *euramericana* I-214, *Fagus sylvatica* L., *Quercus pyrenaica* L., *Paulownia elongata* S.Y.Hu, *Pinus sylvestris* L., and *Picea sitchensis* Trautv. & G.Mey) in cantilever beams exhibited an inversely proportional relationship with the span length of the sample, which results in strong significant differences for the different species.

Taking into account the material used, which is heterogeneous in terms of its organic composition and anisotropic behaviour, the results obtained can be considered highly valuable. The final R^2 value obtained in the global modulus of elasticity of these different species for the prediction model as a function of density and vibration frequency was>95 % and higher than that obtained by ultrasound methods.

This study demonstrates that transversal vibrations in cantilever beams is an extremely precise tool for predicting the static modulus of elasticity of wood and for performing quality control and inspection of wood for structural use. This could be further developed into a useful tool for timber grading.

CRediT authorship contribution statement

Luis Acuña: Conceptualization, Methodology, Investigation, Data curation, Writing – original draft. **Roberto Martínez:** Data curation, Writing – review & editing. **Eleana Spavento:** Methodology, Investigation, Writing – original draft. **Milagros Casado:** Methodology, Supervision, Investigation, Writing – original draft. **Javier Álvarez-Martínez:** Investigation. **Conan O'Ceallaigh:** Investigation, Writing – review & editing. **Annette M. Harte:** Writing – review & editing. **Jose-Antonio Balmori:** Methodology, Investigation, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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References

[1] R.J. Ross, E.A. Geske, G.L. Larson, J.F. Murphy, Transverse vibration nondestructive testing using a personal computer, US Department of Agriculture, Forest Service, Forest Products Laboratory., 1991.
[2] R. Ross, R. Pellerin, Non-destructive Testing for Assessing Wood Members in Structures: A Review. , United States Department of Agriculture. Forest Service. Forest Products. Laboratory General Technical Report FPL-GTR-70, 1994.
[3] R.J. Ross, J.I. Zerbe, X. Wang, D.W. Green, R.F. Pellerin, Stress wave nondestructive evaluation of Douglas-fir peeler cores, For. Prod. J. 55 (2005) 90–94.
[4] F. Divos, T. Tanaka, Relation between static and dynamic modulus of elasticity of wood, Acta Silv. Lignaria Hungarica (2005) 105–110.
[5] L. Acuña, R. Diez, M. Casado, Ultrasounds and the quality of structural timber. Application to Pinus pinaster Ait. [Los ultrasonidos y la calidad de la madera estructural. Aplicación a Pinus pinaster Ait.], Boletín Del CIDEU. 2 (2006) 7–26.
[6] M. Casado, L. Acuña, D. Vecilla, L.A. Basterra, V. Pando, E. Relea, Determinación de la capacidad resistente de madera estructural de Pinus sylvestris mediante PLG., in: XI Congr. Español END. , 2007: pp. 233–242.
[7] S.-Y. Wang, J.-H. Chen, M.-J. Tsai, C.-J. Lin, T.-H. Yang, Grading of softwood lumber using non-destructive techniques, J. Mater. Process. Technol. 208 (2008) 149–158, https://doi.org/10.1016/j.jmatprotec.2007.12.105.
[8] M. Casado, L. Acuña, D. Vecilla, L.A. Basterra, E. Relea, G. López, G. Ramón-Cueto, Técnicas vibratorias aplicadas a madera estructural de Populus x euramericana. , in: V Congr. For. Español, Ávila, 2009.
[9] L.A. Basterra, L. Acuña, M. Casado, G. Ramón-Cueto, G. López, Diagnóstico y análisis de estructuras de madera mediante técnicas no destructivas: aplicación a la Plaza Mayor de Chinchón (Madrid), Inf. La Construcción. 61 (2009) 21–36, https://doi.org/10.3989/ic.09.016.

[10] F. Divos, F. Sismandy Kiss, Strength Grading of Structural Lumber by Portable Lumber Grading - effect of knots, in: Futur. Qual. Control Wood Wood Prod. Final Conf. COST Action E53, Edinburgh, 2010: pp. 4–7.
[11] L. Acuña, L.A. Basterra, M.M. Casado, G. López, G. Ramón-Cueto, E. Relea, C. Martínez, A. González, Aplicación del resistógrafo a la obtención de la densidad y la diferenciación de especies de madera, Mater. Construcción. 61 (2011) 451–464, https://doi.org/10.3989/mc.2010.57610.
[12] M. Casado, L. Acuña, L.-A. Basterra, G. Ramón-Cueto, D. Vecilla, Grading of structural timber of Populus × euramericana clone I-214, Holzforschung 66 (2012) 633–638, https://doi.org/10.1515/hf-2011-0153.
[13] K.C. Schad, D.E. Kretschmann, K.A. McDonald, R.J. Ross, D.A. Green, Stress Wave Techniques for Determining Quality of Dimensional Lumber from Switch Ties, US Department of Agriculture, Forest Products Laboratory, Forest Service, 1995.
[14] Z. Wang, L. Li, M. Gong, Measurement of dynamic modulus of elasticity and damping ratio of wood-based composites using the cantilever beam vibration technique, Constr. Build. Mater. 28 (2012) 831–834, https://doi.org/10.1016/j.conbuildmat.2011.09.001.
[15] E. Baradit, P. Niemz, A. Fernández-Pérez, Propiedades físico-mecánicas de algunas maderas nativas chilenas coníferas y latifoliadas por ultrasonido, Maderas. Cienc. y Tecnol. (2013), https://doi.org/10.4067/S0718-221X2013005000019.
[16] J. Hunt, H. Zhang, Z. Guo, F. Fu, Cantilever beam static and dinamic response comparison with Mid-point bending for thin MDF composite panels, BioResources 8 (2013) 115–129.
[17] M. Bather D. Dirley-Ellis D. Gil-Moreno Combining of results from visual inspection, non-destructive testing and semi-destructive testing to predict the mechanical properties of western hemlock World Conf 2016 Timber Eng., Vienna, Austria.
[18] D.F. Llana, G. Íñiguez-González, J. Montón, F. Arriaga, In-situ density estimation by four nondestructive techniques on Norway spruce from built-in wood structures, Holzforschung 72 (2018) 871–879, https://doi.org/10.1515/hf-2018-0027.
[19] V. Nasir, S. Nourian, S. Avramidis, J. Cool, Stress wave evaluation by accelerometer and acoustic emission sensor for thermally modified wood classification using three types of neural networks, Eur. J. Wood Wood Prod. 77 (2019) 45–55, https://doi.org/10.1007/s00107-018-1373-1.
[20] D.F. Llana, G. Íñiguez-González, M.R. Díez, F. Arriaga, Nondestructive testing used on timber in Spain: A literature review, Maderas. Cienc. y Tecnol. (2020), https://doi.org/10.4067/S0718-221X2020005000201.
[21] Z. Xin, D. Ke, H. Zhang, Y. Yu, F. Liu, Non-destructive evaluating the density and mechanical properties of ancient timber members based on machine learning approach, Constr. Build. Mater. 341 (2022), 127855, https://doi.org/10.1016/j.conbuildmat.2022.127855.
[22] U.B. Halabe, G.M. Bidigalu, H.V.S. GangaRao, R.J. Ross, Nondestructive Evaluation of Green Dimension Lumber Using Stress Wave and Transverse Vibration Techniques, in: Rev. Prog. Quant. Nondestruct. Eval., Springer US, Boston, MA, 1996: pp. 1891–1894. https://doi.org/10.1007/978-1-4613-0383-1_247.
[23] D.W. Haines, J.M. Leban, C. Herbe, Determination of Young's modulus for spruce, fir and isotropic materials by the resonance flexure method with comparisons to static flexure and other dynamic methods, Wood Sci. Technol. 30 (1996) 253–263.
[24] S. Sanchez-Beitia, J. Barrallo, Learning heritage restoration, learning mathematics, in: Alan Rogerson. (Ed.), Int. Conf. Math. Educ. into 21st Century Math. Living, , Amman, Jordania. , 2000.
[25] W.M.G. Burdzik, P.D. Nkwera, Transverse vibration tests for prediction of stiffness and strength properties of full size Eucalyptus grandis, For. Prod. J. 52 (2002) 63–67.
[26] J. Ilic, Dynamic MOE of 55 species using small wood beams, Holz Als Roh- Und Werkst. 61 (2003) 167–172, https://doi.org/10.1007/s00107-003-0367-8.
[27] M. Kisa, Free vibration analysis of a cantilever composite beam with multiple cracks, Compos. Sci. Technol. 64 (2004) 1391–1402, https://doi.org/10.1016/j.compscitech.2003.11.002.
[28] Y. Hu, T. Nakao, T. Nakai, J. Gu, F. Wang, Vibrational properties of wood plastic plywood, J. Wood Sci. 51 (2005) 13–17, https://doi.org/10.1007/s10086-003-0624-9.
[29] Y. Sedik, S. Hamdan, I. Jusoh, M. Hasan, Acoustic Properties of Selected Tropical Wood Species, J. Nondestr. Eval. 29 (2010) 38–42, https://doi.org/10.1007/s10921-010-0063-7.
[30] F. Arriaga, C. Osuna-Sequera, I. Bobadilla, M. Esteban, Prediction of the mechanical properties of timber members in existing structures using the dynamic modulus of elasticity and visual grading parameters, Constr. Build. Mater. 322 (2022), 126512, https://doi.org/10.1016/j.conbuildmat.2022.126512.
[31] D. Gil-Moreno, D. Ridley-Ellis, A.M. Harte, Timber grading potential of Douglas fir in the Republic of Ireland and the UK, Int. Wood Prod. J. 10 (2019) 64–69, https://doi.org/10.1080/20426445.2019.1617984.
[32] K. Simic, V. Gendvilas, C. O'Reilly, A.M. Harte, Predicting structural timber grade-determining properties using acoustic and density measurements on young Sitka spruce trees and logs, Holzforschung 73 (2019) 139–149, https://doi.org/10.1515/hf-2018-0073.
[33] E. Oberhofnerová, K. Arnetová, T. Holeček, V. Borůvka, J. Bomba, Determination of Correlation between Destructive and Nondestructive Test Methods Applied on Modified Wood Exposed to Natural Weathering, BioResources 11 (2016). https://doi.org/10.15376/biores.11.2.5155-5168.
[34] I. Rashidi-Jouybari, P. Lenz, J. Beaulieu, S. Nadeau, J. Bousquet, A. Achim, Multi-trait selection for improved solid wood physical and flexural properties in white spruce, For. An Int. J. For. Res. 95 (2022) 492–503, https://doi.org/10.1093/forestry/cpac006.
[35] E. Goens, Determination of Young's modulus from flexural vibrations, Ann Phys. Ser. 11 (1931) 649–678.
[36] R. Hearmon, The influence of shear and rotary inertia on the free flexural vibration of wooden beams, Br. J. Appl. Phys. 9 (1958) 381–388.
[37] R. Hearmon, Theory of the vibrational testing of wood, For. Prod. J. 16 (1966) 29–40.
[38] M. Merhar, B. Bučar, Determination of correction coefficient for dynamic modulus of elasticity obtained by analysing the frequency response of a clamped cantilever specimen, Holz Als Roh- Und Werkst. 66 (2008) 233–235, https://doi.org/10.1007/s00107-007-0219-z.
[39] S.D. Yu, Free and forced flexural vibration analysis of cantilever plates with attached point mass, J. Sound Vib. 321 (2009) 270–285, https://doi.org/10.1016/j.jsv.2008.09.042.
[40] L. Karlinasari, H. Baihaqi, A. Maddu, T.R. Mardikanto, The Acoustical Properties of Indonesian Hardwood Species, MAKARA Sci. Ser. 16 (2012), https://doi.org/10.7454/mss.v16i2.1405.
[41] D.F. Llana I. Short A.M. Harte Acoustic measurement differences on trees and logs from hardwoods in wet and dry condition in: 21st Int. Nondestruct. Test. Eval. Wood Symp., Freiburg im Breisgau 2019 Germany 561 568.
[42] V. Bucur, R.R. Archer, Elastic constants for wood by an ultrasonic method, Wood Sci. Technol. 18 (1984) 255–265, https://doi.org/10.1007/BF00353361.
[43] J.L. Sandoz, Grading of construction timeber by ultrasound, Wood Sci. Technol. 23 (1989) 95–108, https://doi.org/10.1007/BF00350611.
[44] F.C. Beall, Overview of the use of ultrasonic technologies in research on wood properties, Wood Sci. Technol. 36 (2002) 197–212, https://doi.org/10.1007/s00226-002-0138-4.
[45] U. Dackermann, R. Elsener, J. Li, K. Crews, A comparative study of using static and ultrasonic material testing methods to determine the anisotropic material properties of wood, Constr. Build. Mater. 102 (2016) 963–976, https://doi.org/10.1016/j.conbuildmat.2015.07.195.
[46] C. Osuna-Sequera, D.F. Llana, E. Hermoso, F. Arriaga, Acoustic wave velocity in long pieces of Salzmann pine for in-situ structural assessment, Constr. Build. Mater. 269 (2021), 121256, https://doi.org/10.1016/j.conbuildmat.2020.121256.
[47] M.J. Morales-Conde, J.S. Machado, Evaluation of cross-sectional variation of timber bending modulus of elasticity by stress waves, Constr. Build. Mater. 134 (2017) 617–625, https://doi.org/10.1016/j.conbuildmat.2016.12.188.
[48] D. Ridley-Ellis, D. Gil-Moreno, A.M. Harte, Strength grading of timber in the UK and Ireland in 2021, Int. Wood Prod. J. 13 (2022) 127–136, https://doi.org/10.1080/20426445.2022.2050549.
[49] M. Gürgöze, H. Batan, A note on the vibrations of a restrained cantilever beam carrying a heavy tip body, J. Sound Vib. 106 (1986) 533–536, https://doi.org/10.1016/0022-460X(86)90197-5.
[50] I. Burawska-Kupniewska, P. Mańkowski, S. Krzosek, Mechanical Properties of Machine Stress Graded Sawn Timber Depending on the Log Type, Forests 12 (2021) 532, https://doi.org/10.3390/f12050532.
[51] S.K. Jang, C.W. Bert, Free vibration of stepped beams: exact and numerical solutions, J. Sound Vib. 130 (1989) 342–346.
[52] S.H. Farghaly, Bending vibrations of an axially loaded cantilever beam with an elastically mounted end mass of finite length, J. Sound Vib. 156 (1992) 373–380, https://doi.org/10.1016/0022-460X(92)90706-4.
[53] N.M. Auciello, Transverse vibrations of a linearly tapered cantilever beam with tip mass of rotary inertia and eccentricity, J. Sound Vib. 194 (1996) 25–34, https://doi.org/10.1006/jsvi.1996.0341.
[54] J.R. Banerjee, Explicit frequency equation and mode shapes of a cantilever beam coupled in bending and torsion, J. Sound Vib. 224 (1999) 267–281, https://doi.org/10.1006/jsvi.1999.2194.
[55] D. Negahban Vibrations of cantilever beams: deflection, frequency and research uses 1999 Scott Whitney.
[56] R.M. Digilov, Flexural vibration test of a cantilever beam with a force sensor: fast determination of Young's modulus, Eur. J. Phys. 29 (2008) 589–597, https://doi.org/10.1088/0143-0807/29/3/018.
[57] K.T. Lee, Vibration of two cantilever beams clamped at one end and connected by a rigid body at the other, J. Mech. Sci. Technol. 23 (2009) 358–371, https://doi.org/10.1007/s12206-008-1008-2.
[58] M. Roohnia, An Estimation of Dynamic Modulus of Elasticity in Cantilever Flexural Timber Beams, Drv. Ind. 65 (2014) 3–10, https://doi.org/10.5552/drind.2014.1229.
[59] AENOR, UNE-EN 408:2011. Timber structures. Structural timber and glued laminated timber. Determination of some physical and mechanical properties, Spain, n.d.
[60] AENOR, UNE 56544:2022. Visual grading for structural sawn timber. Coniferous timber., Spain, n.d.
[61] AENOR, UNE 56546:2022. Visual strength grading for structural sawn timber. Hardwood timber., Spain, n.d.
[62] AENOR, UNE-EN 384:2016. Structural timber. Determination of characteristic values of mechanical properties and density, Spain, n.d.
[63] C. Truesdell, Timoshenko's History of Strength of Materials (1953), in: An Idiot's Fugitive Essays Sci, Springer, New York, New York, NY, 1984, pp. 251–253, https://doi.org/10.1007/978-1-4613-8185-3_30.
[64] R Core Team, A language and environment for statistical computing. R Foundation for Statistical Computing, URL Http//Www.R-Project.Org/. (2014).
[65] R. Wilcox, Introduction to robust estimation and hypothesis testing, 5th editio, Academic Press, London, UK, 2021.
[66] B.L. Welch, On the comparison of several mean values: An alternative approach, Biometrika 38 (1951) 330–336.
[67] M. Stone, Cross-Validatory Choice and Assessment of Statistical Predictions, J. R. Stat. Soc. Ser. B 36 (1974) 111–133, https://doi.org/10.1111/j.2517-6161.1974.tb00994.x.

[68] Á. Fernández-Serrano, A. Villasante, Modulus of rupture prediction in Pinus sylvestris with multivariate models constructed with resonance, ultrasound, and wood heterogeneity variables, BioResources 17 (2021) 1106–1119. https://doi.org/10.15376/biores.17.1.1106-1119.
[69] M. Casado L. Acuña E. Caceres Á. Pozo Caracterización físico-mecánica de la madera de Paulownia elongata S.Y. Hu de una plantación del Valle del Duero (Valladolid) in: 7°Congr. For. Español, Sociedad Española de Ciencias Forestales 2017 Plasencia 1 9.
[70] I.R. Kliger, M. Perstorper, G. Johansson, P.J. Pellicane, Quality of timber products from Norway spruce, Wood Sci. Technol. 29 (1995), https://doi.org/10.1007/BF00194198.
[71] R. Pommier, D. Breysse, J.-F. Dumail, Non-destructive grading of green Maritime pine using the vibration method, Eur. J. Wood Wood Prod. 71 (2013) 663–673, https://doi.org/10.1007/s00107-013-0727-y.
[72] K.T.S. Hassan, P. Horáček, J. Tippner, Evaluation of Stiffness and Strength of Scots Pine Wood Using Resonance Frequency and Ultrasonic Techniques, BioResources 8 (2013). https://doi.org/10.15376/biores.8.2.1634-1645.
[73] F. Arriaga, J. Monton, E. Segues, G. Íñiguez-Gonzalez, Determination of the mechanical properties of radiata pine timber by means of longitudinal and transverse vibration methods, Holzforschung 68 (2014) 299–305, https://doi.org/10.1515/hf-2013-0087.
[74] A. Villasante, G. Íñiguez-González, L. Puigdomenech, Comparison of various multivariate models to estimate structural properties by means of non-destructive techniques (NDTs) in *Pinus sylvestris* L. timber, Holzforschung 73 (2019) 331–338, https://doi.org/10.1515/hf-2018-0103.