



PROMOTING THE EVOLUTION OF COOPERATION  
IN SOCIAL DILEMMA GAMES USING CONTINGENT  
MOBILITY STRATEGIES

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*promoting the evolution of cooperation in social dilemma  
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## ABSTRACT

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Researchers in many domains use social dilemmas to explore the conditions necessary for cooperation to emerge among groups. Social dilemmas, and in particular the Prisoner's Dilemma, are used because of their usefulness in capturing the conflict that exists between individually rational decisions and those made to benefit the common good. Evolutionary game theory provides a shared mathematical framework to interpret the evolution of cooperation.

Contingent mobility is strategically driven, purposeful movement that is traditionally achieved through heuristically designed agent strategies or through highly restrictive globally defined conditions. Evolutionary computation techniques are used to develop contingent mobility strategies to promote cooperation without centralised control or costly memories in the Prisoner's Dilemma game.

The evolution of cooperation is promoted by contingent mobility strategies if the strategies facilitate the formation of spatially contiguous clusters of cooperative agents. This thesis presents a mobility model that allows cooperators to pro-actively maintain and grow these spatial structures, which they need in order to become highly fit and, ultimately, survive. A number of cluster metrics are proposed to both quantitatively and qualitatively measure the growth of these cooperative clusters.

A range of evolutionary and environmental settings are investigated, and their effects on the co-evolution of cooperation and mobility are analysed. Variations in population density, agent lifespan, and the birth-death update rule are compared in terms of the levels of cooperation and the evolved mobility strategies they produced. An N-Player extension of the Prisoner's Dilemma is considered to measure the performance of these contingent strategies in more challenging games.

Results from a variety of experiments show that emergent contingent mobility strategies promote cooperation using pure game strategies in the Prisoner's Dilemma. In particular, in a wide range of settings *Follow-Flee*, a strategy proposed and evolved in this work, has been shown to out compete other contemporary mobility strategies, and furthermore, practically guarantee the evolution of cooperation.

## DECLARATION

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I, Maud Gibbons, declare that this thesis is entirely composed of my own work, except where explicitly stated otherwise, and that this work has not been submitted for any other degree or professional qualification.

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Maud Gibbons, B.Sc.

*April, 2019*



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## INTRODUCTION

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The evolution of cooperation is a topic that has garnered attention from a vast array of research fields. Biologists seek to explain the observed behaviour of organisms, social and political scientists analyse the relationships between individual humans and human groups, economists attempt to model economic competition between commercial businesses, and computer scientists wish to simulate these patterns in order to create autonomous, decentralised systems. All of the aforementioned fields desire to study the emergent patterns of groups that arise from the actions taken by competing individuals, and in many cases to construct a system in which rational individuals choose to cooperate.

In the overlapping fields of evolutionary computation and game theory, the goal of rational individuals is to maximise their utility or expected payoff. However, in nature and other competitive systems rational individuals often exhibit altruism, which is loosely defined as actions that benefit the utility of other individuals at a cost of one's own. This phenomenon has been viewed as contradictory because of the inherent conflict between individually rational choices and those made for the common good. Researchers explore the conditions and mechanisms necessary to promote the evolution of cooperation.

### 1.1 INTRODUCTION TO EVOLUTIONARY COMPUTATION

Many of the most successful artificial intelligence (AI) systems built to date have been inspired in some way by nature. Popular fields of research, such as neural networks ([Hopfield, 1982](#)), genetic programming ([Koza, 1992](#)), and swarm intelligence ([Beni and Wang, 1993](#)), look to the solutions of problems found in

the natural world, and attempt to replicate them in order to design intelligent systems. Evolutionary computation is a group of algorithms for global optimization inspired by biological evolution.

The general form of evolutionary algorithms consist of a set or population of agents (or solutions), which are incrementally improved over a number of generations through several processes that resemble natural selection (Holland, 1992a). The main appeal of these algorithms is their ability to solve problems that have a complex solution space or that are hard to model mathematically. For example, evolutionary computation has been applied to social dilemmas in game theory so that agents can learn and adopt strategies from other more successful players.

## 1.2 INTRODUCTION TO GAME THEORY

Game theory is the mathematics of decision making, it describes the strategic interactions that occur between rational agents. Social dilemmas arise when there is a conflict between individually rational or self-interested decisions and those made to benefit the common good. The Prisoner's Dilemma (PD) is ubiquitous in the social dilemmas, and has been extensively studied to explore the conditions necessary for cooperation to emerge among groups or societies of rational agents.

Research in this field began with a large focus on optimal decision making in economics, but as time went by innovative minds recognised that the study of cooperation could be gainfully applied to domains such as psychology (Rapoport et al., 1965), theoretical biology (Trivers, 1971; Smith and Price, 1973), economics (Morgenstern and Von Neumann, 1953), and computer science (Axelrod and Hamilton, 1981). A greater understanding of these mechanisms allows one to make predictions about the behaviour of groups in various scenarios, and for the development of suitable strategies to capitalise on those predicted behaviours.

However, the static ‘solutions’ of game theory, such as the Nash equilibrium (Nash, 1951), obtained by analysing the behaviour of rational agents are fairly unrealistic. Human players, for example, rarely behave with perfect rationality because they don’t have complete knowledge of the game state or can’t compute the optimal move. In many real life environments, agents adapt and change their behaviour over time.

### 1.2.1 *Introduction to Evolutionary Game Theory*

Evolutionary game theory incorporates ideas from evolutionary theory from biology into game theory to provide a common mathematical framework to interpret the evolution of cooperation (Maynard Smith, 1982a; Hofbauer and Sigmund, 1998). The models, inspired by Darwin’s natural selection (Darwin, 1869) and designed by evolutionary computation, are used to analyse the social dilemma described in game theory. This intersection also helped to shed light on the origins of altruistic behaviour; how agents learned to cooperate with one another despite the drive to be selfish.

Traditionally, the models developed in this field are much more expressive as they attempt to replicate some aspect of the real world. The dynamics of strategy change are studied using populations of agents, in repeated interactions, while constrained by a particular static topology, such as a lattice (Nowak and May, 1992). Using this baseline, many additional mechanisms have been proposed to solve the puzzle that is the evolution of cooperation. Most fit within five broad categories: kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection (Nowak, 2006). However, some of these mechanisms bring with them assumptions and limitations that may be inconsistent with the underlying system being modelled.

For example, both direct and indirect reciprocity require a high degree of cognitive ability; the former demands memories of past actions to enact game strategies, such as tit-for-tat

(Axelrod, 1984), while the latter requires the construction of an intricate reputation system (Nowak and Sigmund, 1998). Of particular interest, network reciprocity allows for the creation of ‘network clusters’ (Langer et al., 2008) between unrelated, simple agents without the need for complex reasoning or computation.

### 1.2.2 *Introduction to Mobility*

Agent mobility is a form of network reciprocity (Ohtsuki et al., 2006) that allows agents to respond to their local environment through movement rather than by changing their game strategy. It is of particular interest because it creates a more realistic framework to model a wide range of scenarios including sophisticated human interactions as well as those between microscopic organisms. This movement can occur randomly or deterministically, and can take effect at a local or global level.

Research in this topic has long been hindered by the prevailing notion that individual mobility leads exclusively to widespread defection because of the ‘free rider’ effect (Enquist and Leimar, 1993). This is the phenomenon whereby agents who chose not to cooperate, but instead defect, are able to exploit many more cooperating individuals, or cooperators, and avoid retaliation by moving quickly through their environment. However, recent research has focused on developing movement patterns that promote the creation of clusters, which can shift the balance in favour of the cooperators. Specifically, mobility can allow altruistic players to overcome the temptation to defect by clustering together and to avoid repeated interactions with selfish players.

## 1.3 OPEN QUESTIONS

In recent decades, agent mobility has garnered a lot of attention in terms of its impact on the evolution of cooperation, particularly with regard to purposeful or contingent mobility. It has gone from being perceived as a hindrance to being recognised

as a key factor in the promotion of cooperation in spatial environments. However, a satisfactory contingent mobility model has yet to be adequately described in the literature.

To date, there has been little success in establishing the outbreak of cooperation in highly dynamic environments or using highly mobile populations of agents. Much of the literature suggests or concludes that in order to create conditions suitable for the evolution of cooperation, mobility levels should be kept moderate to minimal. As a result, there is scope to investigate developing contingent mobility strategies to function in these more chaotic environments. Capitalising on a proactive version of agent mobility could be the key to unlocking this puzzle.

Contingent mobility is a more realistic model for agent autonomy than random mobility and diffusion models, but it is generally much more computationally complex and memory intensive. Additionally, many researchers choose mobility in pursuit of pure strategies, which spread cooperation without the need to conditionally defect. As a result, many of the movement algorithms proposed in the literature rely on centralised control mechanisms, which impose heavy information requirements on individual agents. However, in some application domains, such as swarm intelligence and robotics, complex behaviours need to be produced by hardware with limited processing power and memory. In this work, a contingent mobility strategy, named *Follow-Flee*, that fits these criteria is proposed for promoting the evolution of cooperation in the PD.

The majority of mobility mechanisms are heuristically crafted, and require extensive tuning. Often times it can be difficult to understand the motivation or justification for some of these mechanisms. In their review of co-evolutionary games, Perc et al (Perc and Szolnoki, 2010) argue that mobility constitutes a co-evolutionary rule with a promising avenue for future exploitations. However, the evolution of mobility strategies, using genetic algorithms or other evolutionary techniques, has received relatively little attention in the literature.

Finally, another aspect of this domain not fully explored in the literature are the various N-Player extensions to the PD

game. These larger social dilemmas offer further realism to the agents' interactions but generally speaking it is much more difficult to promote the evolution of cooperation within them. These interaction models may act as a litmus test for the success of the contingent mobility strategies developed in this work.

The following open research questions have been identified from the literature that are addressed in this thesis:

1. *Does contingent mobility promote the evolution of cooperation in the Prisoner's Dilemma game with highly dynamic environments due to a highly mobile population of agents?*
2. *Can a contingent mobility strategy, capable of promoting the evolution of cooperation, be constructed using only local environmental information with limited agent complexity?*
3. *Can mobility be co-evolved with cooperation in order to produce an uncomplicated contingent mobility strategy capable of promoting cooperation in dynamic environments?*
4. *Can contingent mobility promote the evolution of cooperation in the N-Player extension of the Prisoner's Dilemma game?*

#### 1.4 HYPOTHESES

1. *Follow-Flee is a viable competitive strategy for promoting cooperation in the spatial Prisoner's Dilemma game.*
2. *Contingent mobility promotes the creation of cooperative clusters of agents playing the Prisoner's Dilemma using pure strategies in a range of environmental and evolutionary settings.*
3. *The formation and growth of cooperative clusters promotes the evolution of cooperation among agents using pure strategies in a range of environmental and evolutionary settings.*
4. *The evolution of cooperation in mobile populations on spatial environments is heavily influenced by a range of environmental and evolutionary settings, including: population density, agent lifespan, and the birth-death update rules.*

## 1.5 RESEARCH CONTRIBUTIONS

In this thesis, the design and implementation of a multi-agent system to simulate the evolution of cooperation in social dilemma games using contingent mobility strategies is presented. Several different representations of agent mobility are constructed based on the agents' perception of their local environment, and a set of actions are designed to facilitate the co-evolution of cooperation and mobility. A ranking function is also developed to calculate the optimal movement locations for each combination of environmental scenarios. The following describes the principal research contributions of this thesis.

1. A survey of the relevant state-of-the-art literature in the domain of Evolutionary Game Theory was completed.
2. An evolutionary model was designed and developed to facilitate the simulation of a population of mobile agents playing the PD on a toroidal lattice grid.
3. A mobility model was designed and developed to represent a wide range of movements while also handling the choices of individual agents in many environment scenarios. Several notable refinements are also presented.
4. A set of cluster metrics were formulated to analyse the creation and growth of cooperative clusters in a toroidal lattice grid using the Moore neighbourhood.
5. Extensive experimentation and an empirical evaluation of the results was performed on each of the models and extended models developed in this work.
6. A contingent mobility strategy titled *Follow-Flee* was presented, compared to a prominent approach from the literature, and evolutionarily verified.
7. Results demonstrating the co-evolution of cooperation and mobility of agents using pure strategies to play the PD in

a range of environmental and evolutionary settings were presented.

8. Results demonstrating the evolution of cooperation in the N-Player PD in various environmental and evolutionary settings using evolved mobility strategies were presented.
9. The finding of this thesis have been disseminated to the research community, peer-reviewed and published at international conferences and as a chapter in a Springer book.

### 1.5.1 Publications

The work in this thesis has appeared in several publications in various venues, as detailed below.

- Gibbons, M.D., O’Riordan, C. and Griffith, J., 2016, August. Follow flee: A contingent mobility strategy for the spatial prisoner’s dilemma. In International Conference on Simulation of Adaptive Behavior (pp. 34-45). Springer, Cham.

Parts of this publication appear in Chapter 3 of this thesis.

- Gibbons, M.D., O’Riordan, C. and Griffith, J., 2016. Evolution of cooperation in N-player social dilemmas: The importance of being mobile. In Proceedings of the 8th International Conference on Computational Intelligence, Vol 1: ECTA. SciTePress Digital Library.

Parts of this publication appear in Chapter 4 of this thesis.

- Gibbons, M.D., O’Riordan, C. and Griffith, J., 2019. Evolution of Mobile Strategies in Social Dilemma Games: An Analysis of Cooperative Cluster Formation. In Computational Intelligence (pp. 87-103). Springer, Cham.

Parts of this publication appear in Chapter 4 of this thesis.

- Gibbons, M.D., O’Riordan, C. and Griffith, J., 2019. The impact of environmental and evolutionary factors on the

emergence of cooperation among evolved mobile agents. In Proceedings of the 11th International Conference on Agents and Artificial Intelligence - Volume 2: ICAART. SciTePress Digital Library.

Parts of this publication appear in Chapter 5 of this thesis.

## 1.6 THESIS STRUCTURE

The structure of this thesis is as follows:

Chapter 2 introduces game theory and evolutionary computation. The traditional player strategies used for the evolution of cooperation are examined and more refined techniques currently being explored to accurately model this phenomena are also evaluated. A wide array of mobility strategies from the literature are categorised and reviewed.

In Chapter 3, a contingent mobility strategy for playing the PD, *Follow-Flee*, is proposed. This strategy's ability to promote the evolution of cooperation is measured in a variety of environmental conditions. A comparison with other mobility strategies identified in the literature is performed.

Chapter 4 introduces the notion of a cooperator cluster in the spatial PD, and describes several measures to both quantitatively and qualitatively evaluate these clusters during their lifetime. This chapter analyses the co-evolution of cooperation and contingent mobility in both the 2-Player and N-Player PD, with particular focus on the levels of cooperation achieved, the evolved mobility strategies, and the levels of clustering.

In Chapter 5, a generalised contingent mobility model is introduced. The goal is to produce a more expressive and flexible model, which can be integrated into the evolutionary process. This chapter also analyses the co-evolution of cooperation and contingent mobility in the 2-Player and N-Player game. A wide range of evolutionary and environmental settings are explored using this extended mobility model to determine their influence the evolution of cooperation.

Finally, Chapter 6 provides a summary of the completed work, the conclusions, and proposes a number of possible directions for future work.

## BACKGROUND

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Chapter 2 introduces game theory, and provides a brief history and introduction to some of the player strategies that have been devised for playing the Prisoner's Dilemma and other games. Evolutionary computation is introduced and its applications to game theory are discussed. Following this is an introduction to evolutionary game theory, outlining the main approaches and environments utilised in these games. Finally, the concept of agent mobility is introduced, and a discussion on how agents learn to increase their immediate payoff and to improve their future payoff through movement is presented.

### 2.1 GAME THEORY

In the 20th century, John Von Neumann set out to establish the principles of rational decision making in his book *Theory of games and economic behaviour* (Morgenstern and Von Neumann, 1953). The basic notions of game theory include players (decision makers), actions (strategies), payoffs (benefits, rewards) and preferences over payoffs (objectives). The field was later improved and popularised by John Nash, who introduced vital concepts such as the *Nash Equilibrium* (Nash et al., 1950), which bears his name, and mixed strategies for non-cooperative games (Nash, 1951). A Nash equilibrium among strategies refers to the scenario where no strategy can unilaterally deviate and obtain a better payoff. These concepts form the basis for modern economic theory, and heavily influence theoretical biology.

Social dilemma games allow players to choose either to be selfish, scoring the maximum payoff, or to contribute for the benefit of the whole society, at a cost to this payoff. The question of cooperation between agents playing these games has at-

	C	D
C	R, R	S, T
D	T, S	P, P

Table 2.1: Formulation of the Game Matrix for the PD. It describes the actions agents may take cooperate (C), or defect (D), where T is the temptation to defect, R is the reward for mutual cooperation, P is the punishment for mutual defection, and S is the suckers' payoff.

tained significant attention from many disparate fields such as sociology (Kollock, 1998), physics (Perc et al., 2017), mathematics (Shakarian et al., 2012), and ecology (Doebeli and Hauert, 2005). This popularity is primarily due their succinct representation of the conflict between individually rational choices and those made for the common good. Examples of these games include the *Snow Drift* game, the *Stag Hunt* game, *Public Goods* games, and the *Prisoner's Dilemma*.

### 2.1.1 The Prisoner's Dilemma

The Prisoner's Dilemma is the most often studied game in this domain. It is described as follows: two players make a choice simultaneously to either cooperate or defect. Mutual cooperation yields a reward R for both participants. However, unilateral defection results in a greater payoff, T, for the defector and a worse payoff, S, for the cooperator. If both players chose to defect, both receive P as a punishment such that:  $T > R > P > S$ . A second constraint,  $2R \geq T + P$ , is usually added for iterated games. The payoffs for agents in a 2-Player game can be represented as an NxN payoff matrix as seen in Table 2.1.

In the iterated form of the PD, the players participate in repeated rounds of the game with the same opponent. A finite number of rounds are played but the exact number is unknown to the players. The iterated game allows for the implementation of iterated strategies, which may change over time, rather than pure strategies, which never change, or mixed strategies, which select actions with a certain probability. This version allows for

the development and incorporation of game strategies that enable players to conditionally cooperate when the previous conventional strategy was to always defect.

Axelrod (1981), in his now famous competitions, discovered the most consistent strategy for playing this variant named Tit for Tat (TFT), in which a player begins by cooperating and then copies the previous action of their opponent. In this way mutual cooperation can be maintained and defection is reciprocated. A set of guidelines based on his finding were also proposed for the development of future strategies, these included:

- Don't be envious;
- Don't be the first to defect;
- Reciprocate both cooperation and defection;
- Don't be too clever.

Many variations of this game exist, which allow researchers to explore questions regarding cooperation using voluntary participation (Hauert and Szabo, 2003), defector punishment (Henrich and Boyd, 2001), social rewards (Wu et al., 2017), using spatially explicit structures, and extend the game further from 2-Player into N-Player interactions, which are discussed in the following sections.

### 2.1.2 *Spatial Game Theory*

In order to construct social dilemmas with a more realistic, and less abstract, translation of the natural societies they model, the games are extended to be explicitly spatial. Many topologies including lattice grids (Nowak and May, 1992), 2D planes (Meloni et al., 2009), small world (Santos et al., 2006), and scale free graphs (Poncela et al., 2009) have been investigated. Spatial models promote the evolution of cooperation by constraining player interactions to these particular structures. Players interact with all those within a predefined radius on the structure, or those with whom they share a connection.

The work by Nowak and May (1992) is often cited as the first example of the evolution of cooperation in a spatially explicit environment. They demonstrated that spatial structure could promote cooperation using persistent dynamic patterns. Additionally, the players used discrete pure strategies, i. e. only cooperate or defect, and maintained some level of cooperation without the need for complex game strategies. This work is extended to include models with spatial irregularities (Nowak et al., 1994), and using the iterated strategies TFT (Nowak et al., 2004). Vainstein et al. (2001) further replicate this success on a diluted lattice, and further demonstrate that stable levels of cooperation could be sustained using this 'disordered' structure.

### 2.1.3 *N-Player Social Dilemmas*

Another interesting extension that has been explored in the literature is that of N-player social dilemmas (Yao and Darwen, 1994). This variant captures a wider set of dilemmas (e.g. donating to charity organisations, environmental issues etc.). However, the problem of avoiding exploitation, or free riders, is more difficult in these larger games (Axelrod and Dion, 1988), and as a result cooperation is harder to promote. Reciprocity is less advantageous, because in order for an agent to punish a defector by defecting in retaliation, that agent must also punish all those that did cooperate.

In these games, N agents interact simultaneously in a combined game, rather than each agent interacting with others individually in N games. Each agent can cooperate or defect, and receives a reward based on the number of cooperators present. Additionally, cooperators incur a cost to interact while defectors do not. In one formalism of the game (Boyd and Richerson, 1988), all players receive a benefit based on the number of cooperators present. Cooperators have to pay a cost. No such cost is borne by defecting players. For instance, let B represent some fixed benefit, N the number of players, c the cost and i the number of cooperators. Participants receive  $(B \times i)/N$ . Cooperators

must pay  $c$  and thus receive a net reward of  $((B \times i)/N) - c$ . This, or similar, formulas have been adopted in several other works (O’Riordan and Sorensen, 2008; Suzuki and Arita, 2003). Additionally, some more recent work has focused on developing strategies for the iterated version of the game (Chiong and Kirley, 2012a; Zeng et al., 2016).

## 2.2 EVOLUTIONARY COMPUTATION

In evolutionary computation, a set or population of candidate solutions or agents are initially randomly generated and then iteratively updated and improved (Holland, 1992a). An agent is a computer system that can perceive its environment, can act independently in order to achieve a goal, and persists over time (Franklin and Graesser, 1996). Each new generation of agents is produced by stochastically removing less desired solutions, and introducing small random changes. In biological terminology, a population of individuals or agents is subjected to natural selection and mutation. As a result, the population will gradually evolve to increase in fitness, in this case the chosen fitness function of the algorithm.

### 2.2.1 Genetic Algorithms

A genetic algorithm (GA) (Holland, 1992b) is a type of heuristic search that mimics the process of natural selection as described by Charles Darwin (1869). A subset of evolutionary computation, genetic algorithms generate solutions to optimization problems using techniques inspired by natural evolution, such as selection, mutation and crossover. Solutions are developed through competition among agents, through successive generations, in a population of solutions to a problem. Agents accumulate a score and acquire fitness, which is necessary to reproduce, by interacting in their environment.

When agents encounter an obstacle or problem in their environment their behaviour is determined by their genetic code. A

gene code, in the field of computer science, is a set of factors that are used to identify solutions to a problem, and these are subject to the evolutionary algorithms. These are usually represented as a binary sequence, i. e. zeros and ones. The length and complexity of a genetic code is completely dependent on the needs of the agents, becoming more complex as the number of actions and scenarios that can occur increase. Typically, these genes are then modified and combined before being passed to the next generation. Evolution in this context can be seen as a population of agents over many generations, through trial and error, determining ‘good’ solutions for their environment.

### 2.2.2 *Evolutionary Game Theory*

Evolutionary game theory began when the ideas from evolutionary theory were incorporated into game theory (Maynard Smith, 1982b). The elegance of this field of research is that it can be used in the design and development of evolutionary robots (André and Nolfi, 2016), while it can also provide new tools toward a greater understanding of complex animal behaviour (Miramontes et al., 2014). In their review of the evolutionary dynamics of biological games (Nowak and Sigmund, 2004), Nowak and Sigmund, argue that evolutionary game theory has become “an essential component of a mathematical and computational approach to biology”.

Evolutionary game theory began with the study of concept know as an evolutionary stable strategy (ESS) (Maynard Smith, 1982b), which refers to a strategy that cannot be successfully invaded by a competing strategy. These models were later augmented with methods to describe the fluctuating distribution of strategies in a population over time, called replicator dynamics (Taylor and Jonker, 1978). Weibull (1997) and Gintis (2000) provide good introductions and reviews of the traditional methods of both ESSs and replicator dynamics.

However, these models require a number of restrictive assumptions, which include a well-mixed population in a non-

spatial environment and that the growth rate of any strategy is proportional to its fitness. Given that calculating the ESS of evolutionary games is more difficult in structured environments, such as random and scale-free graphs (Lieberman et al., 2005), computer simulation has been adopted as a main tool in the research of this domain. Roca et al. (2009) provide a review of the methods to address the effects of temporal fluctuations and spatial correlation which were originally “*neglected in the replicator equation*”. Adami et al. (2016) provide a review of the contemporary methods for using agent-based models in evolutionary game theory.

Spatial structure has generally been shown to promote the evolution of cooperation (Nowak et al., 2010). However, some work has suggested that the selection process may be just as influential, and may also be subject to experimentation and variation. Sánchez et al. (2005) present a removal-duplication process which they demonstrate can produce strong reciprocity in the Ultimatum Game. In this model agents accumulate their payoff, and after a variable number of games the agent with the overall minimum fitness is removed, and a new agent is created by duplicating the one with maximum fitness. Liu et al. (2018) present a weak selection model whereby agents update their strategy based on local information, obtained through interaction rather than by using centralised global information required by traditional replicator equations.

### 2.3 MOBILITY

Movement and migration are key factors in solving the puzzle of the evolution of cooperation. Agents in a population prefer to interact with cooperative players than with those who would try to exploit them. Modelling the mobility of agents playing in social dilemmas has gained more attention in recent decades from researchers in an array of domains; including theoretical biology (Pérez and Janssen, 2015; Burgess et al., 2017), physics (Chen et al., 2011b; Javarone, 2016), robotics (Floreano

and Keller, 2010; Bernard et al., 2016), computer science (Crosby and Pissinou, 2007), and human behaviour (Antonioni et al., 2015).

### 2.3.1 *Traditional Approaches*

Mobility was assumed to hinder the evolution of cooperation. Researchers mostly thought that introducing movement as an ability would lead to the creation of a ‘free rider’, who could invade cooperator clusters and halt the spread of cooperation. These agents would always defect, and could move quickly to exploit those clusters without repercussion. Enquist & Leimar (1993) introduced this notion and present one of the first accounts of mobility in the context of game theory. They discuss an empirical model of a mixed population playing an iterated PD game with partner swapping. In this model, agents could swap partners if their expected utility was unsatisfactory. Several mechanisms to fortify cooperation in the form of social control are introduced, including suspicion and gossip, but conclude that mobility seriously restricts the possibility for cooperation to evolve.

Majeski et al. (1999) present a mobility mechanism to facilitate non-compulsory play, which occurs given a certain probability. Agents will only try to move if they are dissatisfied with their current position, and the worse they are doing the greater the probability of moving increases. While the authors do demonstrate the promotion of cooperation, the level of mobility at which this is achieved is incredibly low. They also stress that too much movement is very detrimental to the development of high levels of cooperation as it leads to the break up of clusters.

Sella and Lachmann (2000) proposed a model that allowed agents to migrate between groups located on a torus grid with a certain probability. Additionally, a single cooperator could also move randomly to a new location, thereby creating a new group. However, the mobility that is granted in this model

only produces the dynamic persistence of cooperation and relied too heavily on mutation and self interaction. Another flaw in this work is that cooperators and defectors are not treated equally and despite this the model can still only support very low levels of migration. In a similar model, Ono et al. (2003) investigate cooperation in the PD using migration between non-spatial groups. A mobility rate  $m$  is used to determine the frequency of movement. In this work, migration is only shown to increase the likelihood of cooperation emerging under a very restrictive set of conditions. Again, the authors assume and conclude that mobility suppresses the evolution of cooperation.

Pepper and Smuts (2002) investigate the conditions under which positive assortment may arise in a plant-foragers game. This is an example of agent mobility in a game other than the PD. Positive assortment is said to occur when individuals of the same type are non-randomly aggregated within the same group, then individuals are, on average, more similar to other members of their own group than to the population at large; it can also be seen as a precursor to cooperator clusters. In the game each cell (plants) produce a certain energy yield and agents (foragers) can either show restraint, leaving half the plant behind, or be greedy, eating it whole. Mobility allows agents to move away from low quality environments by using local environmental feedback. The authors found that all agents tended to leave patches containing mostly greedy foragers and to stay in those with restrained individuals; furthermore positive assortment was achieved by these foragers.

Le Galliard et al. (2005) also begin their study by assuming that mobility and altruism are negatively correlated. The co-evolution of unconditional altruism and a mobility rate is modelled using a population on a social network. However, mobility is shown to improve cooperation in environments where the cost of mobility is high and habitat connectivity is low. The authors conclude that the question of whether mobility promotes cooperation was no longer so clear cut; suggesting the idea that constrained mobility could be essential to explain the origin of altruism.

There is no clear line between this work and that reviewed in subsequent sections other than a gradual change in tone. Many of these models and methodologies are continued, but the general view of mobility, and its uses, slowly shift. It is evident that many of these authors primarily considered movement to only be of benefit to agents with extremely low expected utility. Migration and mobility are only considered at an individual level rather than looking at how a cluster or group may become robust from invasion by defectors. This early work influenced much subsequent research, motivating many to attempt to disprove these assumptions and make improvements.

### 2.3.2 *Random Diffusion*

A shared goal of this field is to describe the evolution of cooperation in as simple terms as possible (Axelrod, 1984). Many of the existing PD strategies, such as TFT, require memories of previous encounters which can be expensive and don't scale as easily. Agents are preferred to have low complexity and to require relatively little processing power.

Researchers are divided in their approach to achieving these goals using mobility. One school of thought is to assume that all movement is random. The appeal of random mobility, also called diffusion, or non-contingent mobility, is that it can be used to describe the simplest possible conditions for the evolution of cooperation. Agents in these models have limited complexity and so can be used to describe altruism in the most basic, cellular level environments. However, this type of mobility sacrifices intelligent decision making. Yang & Wang (2011) investigated non-biased migration lattices, planes, and networks and found that the role of migration was independent of structure. Nonetheless, in this section, random mobility will be discussed in terms of its influence on the evolution of cooperation in the PD on a grid, in continuous space, and in non-spatial groups.

Vainstein et al. (2007) is perhaps one of the most influential papers in this domain. They explore the minimal conditions for sustainable cooperation using a spatially structured population on a diluted lattice with the Von Neumann neighbourhood using unconditional, memoryless strategies with non-contingent movements in the context of the weak PD. In other words, they considered the simplest possible scenario for cooperation. The authors proclaim that cooperation is possible in the presence of mobility when the population density is reduced and that *'intermediate mobilities enhance cooperation!*'. The authors also suggest a rule that would allow agents to take advantage of these findings: *'cooperators attract-defectors repel'*. This work was further extended and improved in the strong PD and other related social dilemma games (Sicardi et al., 2009; Vainstein and Arenzon, 2014). It was found that mobility restores the enhancing factor of the spatial structure for the Snow Drift game, as found in the PD game, where cooperation is usually lower than the fully mixed case. Additionally, they propose that a high mobility rate is more suitable to cooperation at low lattice densities, and that low mobility promotes cooperation at intermediate densities.

Meloni et al. (2009) first introduce a model in which PD players are allowed to move in a two-dimensional plane. The model uses unconditional and continuous movement where individuals can change their direction but their velocity is uniform and constant. The neighbourhood of a given agent consists of all agents in the Euclidean radius of 1 and as a result an agent's neighbourhood, or network, is constantly in flux. Cooperation is observed to be promoted and proliferates throughout the network provided that players do not move too fast and given that cooperation is not too expensive when using the weak PD. The authors conclude that movement of individuals prevents the coexistence of different strategies in the long term. Amor & Fort (2011) offer an extension to this model by including altruistic or social punishment. This mechanism is not treated as a strategy but as an action that players may perform against their partners with a certain probability after each round of the game. The authors found that punishing defectors after only 10% of

the cooperator-defector interactions could introduce a critical advantage for cooperators. Cong et al. (2017) also investigate the interplay between individual mobility and punishment.

Antonioni et al. (2014) also consider a population of agents moving randomly in continuous space playing the PD, Snow Drift, and Stag Hunt games. The authors investigate different strategy update rules, and the effect of players diffusing at different velocities. In this model, the magnitude of the displacement of an agent depends on the density of their neighbourhood, and the authors find that this movement policy promotes cooperation in all of the games parameter space. Other noteworthy findings of this paper include a study that compared the use of the Von Neumann and Moore neighbourhoods concluding that the latter was preferable as it allows cooperators to make more cooperators connections.

Vainstein et al. (2014) take a different approach by giving agents a finite physical mass, or hard cores, and allowing them to evolve this trait. Typically, agents in these structures only occupy a single point, and interact with other agents within a certain radius. A hard core defines a section of the plane, using a radius  $r$ , in which an individual agent exclusively inhabit. Hard cores are shown to hinder movement, the agents spend more time in the same region, and that large displacement become less probable. In this model, using constrained movement and hard cores, cooperators may coexist with defectors if they form a percolating cluster. Suarez et al. (2015) also investigate the interplay between mobility rates and neighbourhood size.

Chen et al. (2011a) model collective motion on a 2D plane, which moves from a state of chaos to one of static structure. Each agent is assigned a pure strategy, an angle of direction, and moves with a fixed velocity. At each time step, the players are split, according to periodic boundary conditions, into disconnected groups within which players interact. The strategy of the best performing agent is adopted by its neighbours and agents adopt the average angle of direction of the group. A network of player connections is gradually developed and then becomes static for the duration of the simulation. The au-

thors suggest that cooperation is best achieved with low levels of migration such that cooperator clusters can expand.

Random mobility can also have a major impact in a non-spatial environment as Killingback et al. (2006) demonstrate with their work on the PD as well as public goods games. The groups are well-mixed with each individual playing with every other member of the group. Following interaction and reproduction a certain fraction of the individuals in each group randomly disperse between the other groups. The authors establish that high levels of cooperation are sustained when the group size is small. Janssen and Goldstone later (2006) improve upon this work by enforcing the strong PD condition, and by demonstrating that low levels of mutation and migration are essential in these environments to promote the evolution of cooperation.

### 2.3.3 *Contingent Mobility*

Any movement mechanic that imbues agents with influence over the choice of their new location essentially defines contingent mobility. The major appeal of this type of strategy driven movement is that they offer a more realistic framework for the evolution of cooperation than the random models. While some approaches contain probabilistic or random elements the agents themselves actually make decisions about their environment. On the other hand, the main criticisms of these types of models is that very they often suffer from high complexity and are computationally expensive. A trade-off exists between modelling intelligent decision making and designing agents with limited complexity. Contingent mobility models can largely be divided into those that react to their current neighbourhood and those that actively seek out better locations. In this section we will be examining these contingent movement models and their subtle differences.

Aktipis (2004) in her seminal paper presents a contingent movement strategy for playing the spatial PD. Here, the agents

employ the simple movement rule *Walk Away* to disconnect from defecting partners relocating to a local random cell. Agents form pairs when they meet in the environment, which is quite discordant with contemporary and subsequent environments. The strategy allows cooperators to take advantage of cooperation of mobility rather than it being only used by defectors. The main appeal of this strategy is in its simplicity, agents are memoryless but the Walk Away strategy is still sufficient for cooperation to spread and dominate. In this paper, the strategy is tested and proved effective against itself, TFT, and a spatial version of win stay lose shift. The key behind its success is that this form of mobility allows agents to avoid repeated interactions with defectors and maintain links with other cooperators without employing complex strategies. Here, cooperators become resistant to invasion by defectors without the need for large scale clustering. This work was later extended and applied to public goods games (Aktipis, 2011), and examined as a method of optional participation (Joyce et al., 2006).

Hamilton and Taborsky (2005) a few years later develop a similar ‘win stay, lose shift’ migration rule between groups where agents interact in pairs and are evolved using genetic algorithms. Many mobility strategies emerge but the PAVLOV strategy performs best in terms of facilitating the emergence of cooperation. The authors demonstrate co-evolution of contingent group leaving and conditional cooperation in a non-spatial environment where mobility involves individual agents switching between groups. The agents do not require complex cognitive capabilities as they only need to keep track of their most recent experience to make a decision about moving. However, the movement of agents once it has been initiated is random; agents do not choose which group they will be next placed in.

Yang et al. (2010) present a model based on ‘aspiration induced migration’. Agents move if their current payoff is lower than a predefined aspiration level. Locations for migration are chosen randomly from empty spaces in the agent’s Von Neumann neighbourhood. In this model, agents choose when to move but not where. The authors report that moderate aspira-

tion levels enable cooperator clusters to maintain cooperation and expand while inducing defector clusters to disintegrate. This model is further extended (Lin et al., 2011) to investigate the influence of varying the interaction radius of the agents, which should be moderate in order to produce the highest levels of cooperation.

Roca and Helbing (2011) also prescribe to the idea that players have an aspiration level that determines their satisfaction. However, in this model the aspiration level is both individual and modifiable. Agents in this environment play a public goods game with their neighbours and if agents are dissatisfied with their current payoff they can change strategy or move within a specific radius. An advantage of this approach highlighted by the authors is the relatively low complexity requirements for agents; they have a decaying memory of past interactions as opposed to a full history. It is reported that the best results for a cooperative outcome occur when agents are moderately greedy i.e. when mobility is moderate.

Ichinose et al. (2015) continue the trend of using aspiration based contingent movement and include in the model tag-mediated cooperation. Each agent is assigned one of two arbitrary tags and is given a preference to either always cooperate with those agents with the same tag or those with the opposite tag. This preference for tags is an evolvable trait and is used as the threshold for individual agent migration rather than the actual or expected payoff. Agents move if they are dissatisfied with the tags in their current location but there is no notion of exploration; an agent without any neighbours will not move. The authors observe that cooperation emerges when agents learn to move and to restrict their cooperative attitude to others sharing their same identity. In effect the introduction of tags to aspiration based mobility provides a new route for cooperators to segregate themselves from defectors.

An aspiration based model that uses payoff as a threshold for migration can be more simply viewed as a derivative of calculating the ratio of cooperator to defector strategies in one's neighbourhood. This is the approach taken by Jiang et al. (2010) in

their model titled Adaptive Migration, which only uses the local information obtained through game interactions with their neighbours and as a result has low complexity and computation requirements. Cooperation emerges as agents learn to flee defecting partners, and to remain next to those who cooperate. Cooperation in both the PD and Snow Drift games is shown to become dominant due to the appearance of empty sites between defectors as a result of migration; these defectors cannot gain enough profits from their interactions and become vulnerable to invasion.

Tomassini and Antonioni (2015) also take this approach to mobility in their model. It deals with memoryless agents who interact in pairs with neighbours in continuous space, and proposes an improvement to random diffusion for the displacement of individuals. Lévy flights are a power law distribution strategy such that individuals will migrate locally with a high probability but will displace globally with a low probability i. e. a decay function that leads to a hindered random movement. These Lévy flights are used by individuals as a reaction to growing numbers of defectors in their neighbourhood. The authors find that when agents learn by imitating their most successful neighbour hindered random mobility promotes cooperation in the entire games parameter space. Additionally the authors study both the Von Neumann and Moore neighbourhood settings and conclude that the Moore neighbourhood is more beneficial to the emergence of cooperation as it allows cooperators to establish more cooperator connections.

Perez and Janssen (2015) discuss a resource dynamics game in which movement costs energy, and both forms of mobility are implemented. Agents dissatisfied with their payoff move to the nearest best patch, but may also move randomly with a fixed probability. The authors report a long-term sustainability of the resource when agent mobility is moderate even in harsh environments. Ribeiro de Andrade et al. (2009) also investigate satisfaction as a means to initiate agent mobility.

Cong et al. (2012) present a reputation based migration model, in which agents accumulate a reputation score through previ-

ous cooperative behaviour. Similar to models that employ aspiration levels, agents will move, with a certain probability, away from areas whose agents have low reputations and towards areas with higher reputations. When chosen to move, agents explore their Moore neighbourhood in search of better locations but will only interact with those agents in their Von Neumann neighbourhood. In general, agents with a high reputation score have a better chance of moving; this seems biased towards cooperators and results in isolated individuals remaining isolated. The cooperation that is achieved in this work favours large population densities when the temptation to defect is low and is inhibited by high mobility. In this case, reputation and mobility are too closely intertwined, and the model itself is surprising complex. Both the memory of past interactions and the evaluation of potential new sites are employed and significantly increase the complexity of this implementation.

The works by Helbing and Yu [Helbing and Yu \(2008, 2009\)](#) describe a form of contingent movement called Success Driven Migration (SDM), and together form one of the most influential and important ideas within the domain of mobility. New mechanisms and extensions are proposed for this model nearly every year ([Li et al., 2019](#)). In this model, agents can test potential sites for migration, both local and global, in order to discover neighbourhoods with the highest expected payoff. To achieve this, agents use fictitious play at a negligible, but comparable, cost to themselves. The authors demonstrate that cooperators are improved by migration as it allows them to find other cooperators and avoid defectors creating so called 'islands' or clusters. Additionally, cooperation is shown to emerge for a range of mobility rates and population densities. The main appeal of SDM lies in its ability to get cooperation established and its realism; it has a better narrative for real-world migration than diffusion or random models. However as a result the model's complexity is driven up significantly requiring substantially more computation.

The latter paper tackles the emergence of cooperation under noisy conditions using SDM. The authors attempt to degrade

the model by including random strategy mutations and relocations with the aim of learning whether predominant cooperation can emerge and survive under adverse conditions. It does, and with great success. SDM is shown to generate spatial correlations between cooperators faster than the noise or defectors can destroy them and gives cooperative clusters the ability to regroup following invasion or dispersal.

Buesser et al. (2013) offer an extension to the SDM model that investigates systematically both the interaction and migration radii; these parameters were kept constant in the original model. Furthermore agents are assigned a certain energy level to determine the frequency with which one can search for and test new locations. The authors reveal that widespread cooperation is best obtained when agents interact locally in a relatively small neighbourhood. It was also found that the migration mechanisms were more influential on the prevalence of cooperation than the strategy update rule. This is significant because in traditional immobile models the spread of cooperation is highly dependant on the strategy update rule.

Droz et al. (2009) propose a model for the spatial PD in which certain players have more influence over others in passing on their strategy, and it is only these individuals who can migrate. Influential players move randomly at an optimal rate. The authors report that the highest levels of cooperation are achieved when the model supports a small number of influential players among a large population. However this model is entirely dependant on the actions of influential cooperators and defectors who appear to have an unrealistic and unfair advantage.

#### 2.3.4 *Mobility in the N-Player Prisoner Dilemma*

As research in mobility is relatively new to this domain there are fewer studies focusing on the N-Player PD. Research into mutli-player games and mobility tend to be more focused on public good games. In the following section, papers that explore this intersection are reviewed.

Suzuki and Kimura [Suzuki and Kimura \(2011\)](#) describe a co-evolutionary model for cooperation and random mobility using a population of agents playing the N-Player PD on a dilute lattice grid. Agents move according to an individual mobility rate, which is inherited along with their game strategy (C or D) from their parent. Agents are shown to evolve different mobility rates dependent on their strategy: defectors evolve faster mobility rates than cooperators, who prefer to move slowly to maintain their cooperative connections.

Chiong and Kirley [\(2012b\)](#) describe a random mobility model where a population of agents interact in an N-player PD set in a fully occupied regular lattice. Mobility in this environment is a probability function based on the time you've spent in a location, and the relative fitness of the agent at the destination, and occurs within a certain radius. The agents have a limited memory of past interactions, and past cooperator and defector levels. The authors assume that an agent who has been at a location for an extended period of time will want to move; idleness is the only criteria for movement. Cooperation is shown to be promoted though strictly only under a small set of parameters; the cost to benefit ratio and the range of movement. This model is later applied to agents in an N-Player Snowdrift Game ([Chiong and Kirley, 2013](#)).

Ichinose et al. [\(2013\)](#) propose a model for playing the N-player PD in which agents evolve a probability vector for movement in four directions. The mobility evolved favours directional migration, preferring one direction to be taken but allowing the others, and the authors claim that this shows a collective chasing behaviour between cooperators and defectors. However, the model implemented is probabilistic and the authors make inferences about the emergent patterns that imply decision making, which is a characteristic of deterministic models. Furthermore, upon closer inspection the model relies heavily on mutation; the chasing behaviour cannot emerge without mutation of defectors into cooperators.

Most recently, Suarez et al. [\(2015\)](#) present a contingent mobility model, using the N-Player game, in which agents move to-

ward locations with higher potential payoff. While cooperation does emerge, the authors do not elaborate on the specific effects of mobility, focusing on the impact of the neighbourhood size.

#### 2.3.4.1 *Summary of Mobility*

The role of mobility in the evolution of cooperation has grown in importance over the years spanned by this review. It has gone from being perceived as a hindrance to the emergence of cooperation to its champion. While unrestrained movement can lead to the 'free rider' effect, simple strategy rules or mobility rates significantly curb this phenomenon allowing self-preserving cooperator clusters to form and cooperation to proliferate. Researchers have developed several mechanisms for the emergence of cooperation but they all essentially express a need for cooperators to avoid defector interactions or increase those with cooperators in order to overcome the temptation to defect. Vainsetin et al (2007) summarise this heuristic with a simple rule "Cooperators attract - Defectors repel".

Several environment variables that are amiable to the emergence of cooperation come to the fore in the literature. Sicardi et al. (2009) states that it is possible for cooperation to emerge when the strong form of the PD is in place. Buesser et al. (2013) suggest that cooperation is best achieved when agents interact locally in a relatively small neighbourhood to create cooperator clusters and limit the influence of outside defectors. Antonioni et al. (2014) determine that using the Moore Neighbourhood configuration is preferable to the Von Neumann neighbourhood as dynamics game allows for agents to establish cooperator connections. Vanistein et al. (2014) show that in environments with high mobility cooperation is best achieved when the density of the population on the world is low to allow cooperator clusters to hide from invading defectors.

## 2.4 SUMMARY

In this chapter, the domains of game theory and evolutionary computation were defined, and the concepts central to the research in this thesis, evolutionary game theory and agent mobility are discussed in detail with regard to prominent methodologies in the literature. A comprehensive review of agent mobility models is presented, which defines the broad categories of random diffusion and contingent mobility, and a number of gaps in the literature are identified.

There is clear evidence that there is need for a contingent mobility strategy based on both Axelrod's and Veinstein's rules, which can also handle highly dynamic environments. There is a consistent trend in the literature; too much movement leads to the break up of cooperator clusters and the eventual spread of defection despite the efforts to counteract the 'free rider' effect.



## FOLLOW-FLEE: A CONTINGENT MOBILITY STRATEGY FOR THE PRISONER'S DILEMMA

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In this chapter, a contingent mobility strategy is defined for playing the spatial Prisoner's Dilemma game. The *Follow-Flee* mobility strategy allows agents to increase their percentage of mutually cooperative interactions by pursuing other cooperators and avoiding defectors. Specifically, it allows players to form and sustain clusters by following nearby cooperators, and by fleeing from invading defectors. An evolutionary approach is adopted whereby agents accumulate fitness by playing the game with their immediate neighbours; using a birth-death update rule, where the fittest agents reproduce and replace those with lowest fitness.

Results are presented from a series of simulations comparing the performances of the *Follow-Flee* strategy to *Walk Away*, a heavily cited contingent mobility strategy proposed by Aktipis (2004). The strategies are evaluated and compared in terms of their ability to promote the evolution of cooperation when competing against a *Naïve* (or random) strategy, and when in direct competition with each other.

### 3.1 METHODOLOGY

In the following sections, the environment topology, agent representation, game parameters, the mobility strategies, evolutionary dynamics, and the simulation process used in this work are described. Much of this methodology is common to the entire thesis, and so is referred to in subsequent chapters.

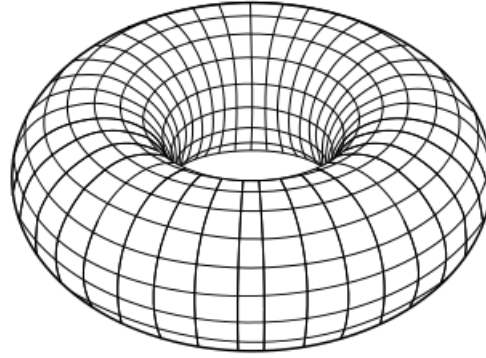


Figure 3.1: A graphical representation of an  $L \times L$  square lattice with periodic boundary conditions. In this structure each site (or cell) is either empty or occupied by one individual.

### 3.1.1 *Environment Topology*

This work considers a population of agents inhabiting a toroidal shaped diluted lattice (see Figure 3.1) with  $L \times L$  cells, each of which can be occupied by up to one agent. This has been the preferred environmental topology adopted in the literature since it first garnered attention by Nowak and May (1992) in the overlapping domains of spatial and evolutionary game theory. The interaction and movement radii of agents are determined using the Moore neighbourhood of radius one. This comprises the eight cells surrounding an agent on the lattice grid. The agents can only perceive and play with those within this radius. These boundary conditions allow for a wider range of movement and present a harder evolutionary problem to solve than the more traditional von Neumann neighbourhood (4 adjacent neighbours).

In this chapter, in order to conduct a fair comparison between the two mobility strategies in question, much of the methodology presented by Aktipis (2004) is replicated. For example, the same values for the population size,  $N$ , and the grid size,  $L$ , are used. However, this work deviates from the Aktipis methodology, in that, the restriction of one agent per cell is enforced, and the interaction radius of agents is expanded. In that work, when two agents occupied the same cell they became paired,

	C	D
C	3, 3	0, 5
D	5, 0	1, 1

Table 3.1: Values for the Game Matrix of the PD. Cooperation (C), or defection (D), with  $T = 5$  the temptation to defect,  $R = 3$  the reward for mutual cooperation,  $P = 1$  the punishment for mutual defection, and  $S = 0$  the suckers' payoff.

and could only interact with that partner. Furthermore, these pairings could only be broken up by other agents moving to the same cell, in which case a new pairing would be created randomly from the three agents in that cell. Multiple pairing could exist at a single cell location but they could not interact with each other. These particular rules used by Aktipis were not adopted because they deviate significantly from contemporary methodologies and arguably are not properly justified as they confer a large advantage to any two cooperators who are placed in the same cell, because they can't be easily invaded.

### 3.1.2 Agent Representation

Each agent in the population is characterized by two different attributes: game strategy and a mobility strategy. The classical version of the PD game is adopted as the interaction model for the agents in our population, in this way, an agent can either cooperate (C) or defect (D). Accordingly, an agent may receive a reward  $R = 3$  for mutual cooperation,  $T = 5$  for successful defection, a punishment  $P = 1$  for mutual defection or  $S = 0$  for exploited cooperation (see Table 3.1).

At each time step, agents participate in a single round of the PD game with each of their neighbours, if any. Agents play using pure strategies; either always cooperate or always defect. Pure strategies are implemented in order to reduce the strategy space allowing us to examine the effect of mobility in these experiments more clearly. Agents are aware of the actions taken by their neighbours in a single round, but these memories do

not persist. This is done to allow agents to accurately identify the strategies of their neighbours when determining their next movement. The payoffs agents receive from playing the game are accumulated and used as their score. The fitness function used in this work is based on these accumulated payoffs within a generation, as we wish to capture both the payoffs and frequency of interaction for individual agents.

### 3.1.3 *Mobility Strategies*

Following the interaction phase, agents then have the opportunity to respond to those interactions by moving to a position in their neighbourhood determined by their mobility strategy. The mobility strategies examined in this chapter include: *Walk Away*, *Follow-Flee*, and *Naïve*.

*Walk Away* is a contingent mobility strategy described by Aktipis (2004) consisting of three simple rules based on an individual agent's environmental state. As outlined by the author, this strategy enables agents to engage in repeated interactions with cooperators and avoid repeated interactions with defectors without using memory. These rules include:

1. If you have no partner, take one step on the grid in a random direction.
2. If your partner defected in the last round of the game, move one step away from them on the grid.
3. If your partner cooperated in the last round, stay in your current position on the grid.

*Follow-Flee* expands upon the version of contingent mobility as characterised by *Walk Away*, i. e. movement rules that consider local information obtained in the last round of the game. However, this strategy more explicitly maintains mutually cooperative interactions, and accounts for a wider range of scenarios than what was possible in the Aktipis methodology. It consists of a number of simple rules, including:

1. If you have no neighbours, take one step on the grid in a random direction.
2. If your neighbour(s) defected in the last round of the game, move one step away from them on the grid.
3. If your neighbour(s) cooperated in the last round, move one step next to them on the grid. This may include your current position.
4. If you have both cooperators and defectors in your neighbourhood, move one step on the grid that satisfies both rules 2 & 3.

*Naïve* agents move one step in a random direction to a free space on the grid regardless of any neighbours.

The mobility strategies chose new locations for agents to move to by evaluating the composition of its current neighbourhood. Each free space on the lattice grid within this area is assigned a score based on the rules specific to each mobility strategy. Neighbourhoods containing multiple agents are evaluated by first calculating the scores for each location per agent, and then accumulating them into a combined score. This in turn is used to rank those available locations.

- *Walk Away* penalises each location adjacent to a defector.
- *Follow-Flee* penalises each location adjacent to a defector, and rewards each location adjacent to a cooperator.
- *Naïve* randomly chooses a new location from the available free spaces.

For example, given an agent **X** using the *Follow-Flee* mobility strategy, Table 3.2 outlines the results of agent **X**'s movement locations being scored in each of the non-trivial scenarios.

In Table 3.2 (left), agent **X** sees a cooperator **C** and adjacent cells are rewarded. The current location is treated as adjacent, thus staying still, or not moving, is a valid option. The opposite is true in Table 3.2 (middle), where agent **X** sees a defector **D**,

<b>C</b>	1	0	1	1	1	<b>C</b>	2	1
1	<b>X (1)</b>	0	0	<b>X (0)</b>	0	1	<b>X (1)</b>	0
0	0	0	0	<b>D</b>	0	0	<b>D</b>	0

Table 3.2: An illustration of how to determine the move performed by an agent using *Follow-Flee* where: all neighbours are cooperators (left); all neighbours are defectors (middle) and neighbours are both cooperators and defectors (right).

adjacent locations score nothing and distant cells are rewarded. In Table 3.2 (right), agent **X** sees both **C** and **D**, multiple neighbours are handled by first calculating a score set for each individual and then combining them. Agent **X** will then move to the location represented by the highest scoring cell, and in the case of a tie, a location is chosen randomly from those cells.

### 3.1.4 The Evolutionary Process

In this work, a birth-death update rule is considered for the evolutionary process rather than the more traditional replicator equations or a natural selection styled model as seen in the literature. Selection is the primary contributor to the development of mobility strategies and emergence of cooperation. The population isn't evolved in the traditional GA sense. However, similar approaches are adopted to study in the fields of theoretical biology and statistical mechanics, as outlined in Chapter 2, where this selection process is considered sufficient. Other methodologies, such as Sanchez and Cuesta's (2005) removal-duplication process, and Aktipis' energy model, are also present in the literature. This type of evolutionary process makes it faster and easier to analyse and visualise the fluctuation effect of varying the individual parameters without the noise typical of the standard evolutionary processes.

At the end of each generation,  $s$  steps of interaction and movement, the population of agents is ranked according to their accumulated fitness score. The highest scoring agents duplicate themselves, placing the offspring randomly on the grid, and the lowest scoring agents die. In this way the population

Variable	Description	Value
Population Size	The total number of agents in a population.	$N = 100$
Simulation Length	The maximum number of time steps in a simulation.	$t = 1000$
Generation Length	The number of turns/steps agents take per generation.	$s = 15$
Replacement Rate	The percentage of the agent population that both dies and reproduces each generation.	$r = 25$
Grid Density (D)	$D = N/L^2$ where L is the length of the lattice grid.	$L = 25$

Table 3.3: The list of evolutionary and environmental variables and their baseline values used for each set of experiments.

density remains constant throughout a simulation. The number of agents replaced in this way is controlled by the replacement rate,  $r$ . At the end of each generation, the fitness score of the whole population is reset. No other genetic operators are used. This evolutionary approach preserves the spatial structure of clusters present in the population across generations.

### 3.1.5 Experimental Setup

A single simulation consists of a population of  $N$  agents placed randomly on a  $L \times L$  torus. A simulation consists of agents taking  $s$  steps per generation, using the replacement rate,  $r$ , over  $t = 1000$  time steps. The game strategies (whether to cooperate or to defect) and the movement strategies are assigned in equal proportion. The distribution of spatial strategies, level of cooperation, the time taken for the simulation to converge on cooperation (or defection), and the total number of interactions are recorded. A total of 3000 simulations are performed for each configuration of density and evolutionary settings. Table 3.3 provides a full list of variables for each set of experiments. These settings and values were chosen because they closely reflect those used by Aktipis (2004).

### 3.2 SIMULATION RESULTS

In this section, the experimental results of the simulations of the PD game on a diluted toroidal lattice grid comparing and contrasting the performances of the mobility strategies *Follow-Flee*, *Walk Away* and *Naïve* are presented. The first set of experiments, consists of both the *Follow-Flee* and *Walk Away* strategies competing separately against the *Naïve* strategy. The strategies are evaluated in terms of their ability to promote the spread of cooperation. Secondly, *Follow-Flee* and *Walk Away* compete where both the number of time steps per generation ( $s$ ), and the reproduction rate ( $r$ ), are varied over a range of values. The third set of experiments compare the relative success of both mobility strategies, competing against the *Naïve* strategy, at different density levels (varying  $L$ ) to investigate the effect, if any, of density on the outcome of a simulation. Finally, both the *Follow-Flee* and *Walk Away* strategies are directly compared, by having them compete in the same simulation without the influence of the *Naïve* strategy.

#### 3.2.1 The Baseline Experiment

To begin, the *Follow-Flee* and *Walk Away* contingent mobility strategies compete separately against the *Naïve* strategy to determine their ability to promote the evolution of cooperation in the PD game. The generation length  $s = 15$  and replacement rate  $r = 25$ , which constitute the lifespan of agents, are fixed, using the grid size of  $L = 25$ .

As is shown in Figure 3.2, the *Follow-Flee* mobility strategy significantly outperforms the *Walk Away* strategy in terms of promoting the evolution of cooperation. The *Walk Away* strategy is only able to promote cooperation in 65% of simulations, while in direct competition with the *Naïve* strategy, whereas the *Follow-Flee* strategy leads to cooperative outcomes in 99% of the simulations. This relatively low level of cooperation achieved by *Walk Away* is surprising, because in the original work Aktipis'

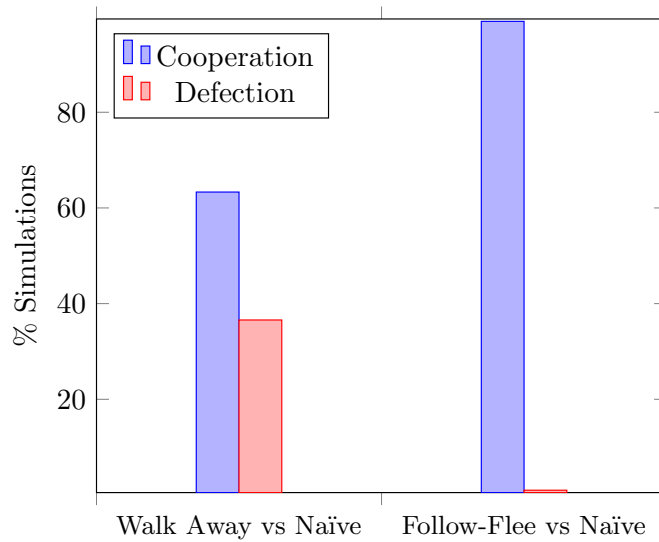


Figure 3.2: Average percentage of simulations resulting in Cooperation (blue), or Defection (red), for the strategies, *Follow-Flee* and *Walk Away*, competing against the *Naïve* strategy, using  $L = 25$ ,  $r = 25$ , and  $s = 15$ .

Strategy	Convergence Speed	Cooperator Interactions
<i>Walk Away</i>	202 timesteps	328,000
<i>Follow-Flee</i>	380 timesteps	382,000

Table 3.4: The convergence speed and number of mutually cooperative interactions generated by each of the two mobility strategies in a single, though typical, simulation which resulted in the evolution of cooperation.

strategy achieved cooperator dominance in 100% of simulations against a similar naïve strategy.

This disparity between these two sets of results may in part be due to the environmental changes introduced in this work; here the restriction of one agent per cell is enforced and agents interact within the Moore neighbourhood. This modification perhaps demonstrates the importance of this constraint on inducing cooperation in the Aktipis work.

Table 3.4 shows that simulations using the *Walk Away* mobility strategy typically converge on a solution more quickly than those using *Follow-Flee*. On the other hand, the *Follow-Flee* mobility strategy generates on average 15% more mutually cooperative interactions than the *Walk Away* strategy. This is sig-

nificant because when defectors are more prevalent, such as in the early states of a simulation, cooperators are under more severe evolutionary pressure, which is alleviated by participating in a larger proportion of beneficial interactions.

### 3.2.2 Varying the Evolutionary Settings

In this experiment, the generation length  $s$  and replacement rate  $r$ , which constitute the lifespan of agents, are both varied while testing the success of both *Follow-Flee* and *Walk Away* as in the previous set up. Success is measured in terms of the strategy's ability to induce cooperation among the population.

The values of  $s$  and  $r$  are varied in order to investigate their impact on the evolution of cooperation using both mobility strategies. Additionally, this investigation should help identify which combination of variables most effectively promote cooperation, which can be used in subsequent experiments. For each pair of values a set of simulations are carried out, as per Experiment 1, while independently varying both values from 5 to 35, in intervals of 5, with a fixed population using  $s$ . In each simulation, the agents will either take an increased number of steps per generation or a larger proportion of the population will participate in the evolutionary process.

Figure 3.3 and 3.4 show the percentage of simulations that end with total adoption of cooperation, i.e. cooperator victories, as the evolutionary parameters,  $r$  and  $s$ , are varied. Simulations always converge on either total cooperation or defection, 'draws' are very rare and only occur in 0.1% of simulations. Across the majority of the parameter space, the *Follow-Flee* mobility strategy outperforms the *Walk Away* strategy in terms of promoting the evolution of cooperation. *Walk Away* has more success in spreading cooperation at lowest values of  $s$ , but across the remainder of the space it performs relatively poorly. When using the most favourable evolutionary settings, only 60% of simulations using the *Walk Away* strategy achieves wide-spread cooperation. On the other hand, *Follow-Flee* dra-

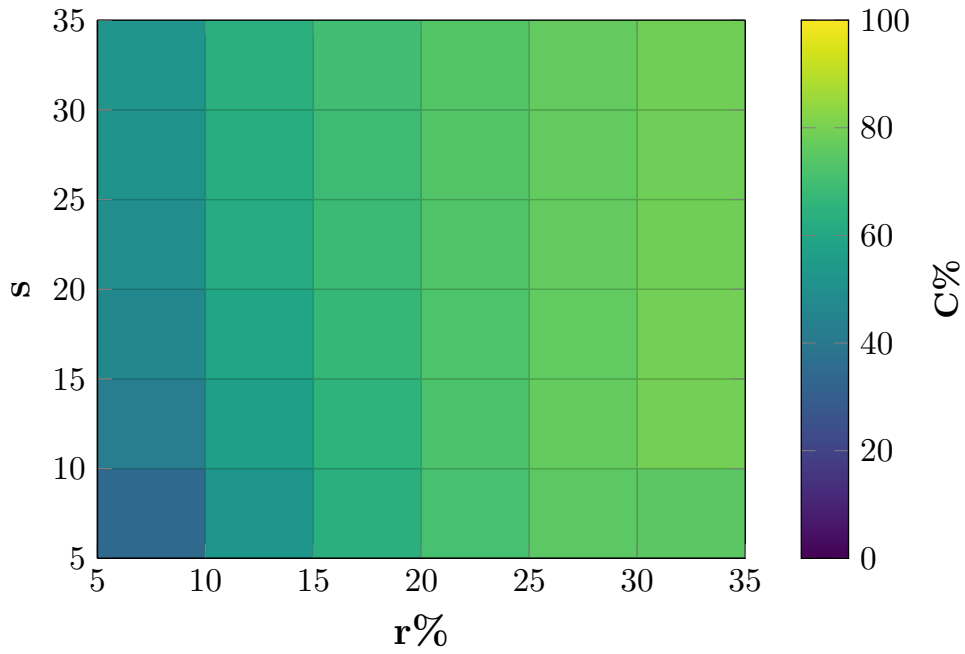


Figure 3.3: Average percentage of cooperator victories for the *Walk Away* mobility strategy for grid size  $L = 25$  as a function of replacement rate  $r$ , and the steps per generation  $s$ .

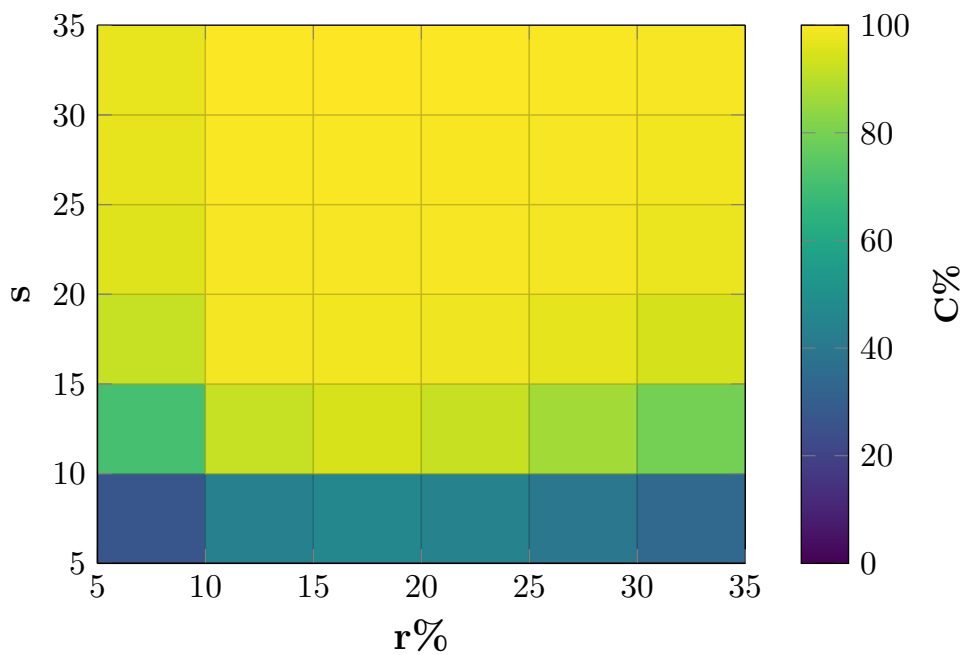


Figure 3.4: Average percentage of cooperator victories for the *Follow-Flee* mobility strategy for grid size  $L = 25$  as a function of replacement rate  $r$ , and the steps per generation  $s$ .

matically improves upon its poor performance when using low values for  $r$  and  $s$ , by promoting the evolution of cooperation in nearly 100% of simulations; managing to almost completely counteract the influence of defectors.

Additionally, it has been shown that the reproduction rate  $r$  has a bigger impact on the outcome of a simulation using the *Walk Away* strategy, while the generation length,  $s$ , has a bigger impact on the outcome of a simulation using the *Follow-Flee* mobility strategy. Both parameters need to be considered in order to produce the best results for the evolution of cooperation.

### 3.2.3 Varying Population Density

In this experiment, the influence of density on the evolution of cooperation using both the *Walk Away* and *Follow-Flee* strategies is investigated. Density is varied by changing the size of the environment rather than the size of the population. This is done to to minimise any unintended impacts these changes may have on the evolutionary process, which is varied and analysed in other experiments.

For each grid density in the range  $L = 20$  to  $L = 60$ , in intervals of 5, a set of simulations are carried out using the values for  $N$ ,  $s$  and  $r$  outlined in Table 3.3. In this way, the performance of the strategies is investigated in both very high and very low densities. A set of simulations is run for both strategies in competition against the *Naïve* strategy.

Figure 3.5 illustrates the relationship between population density and cooperator victories for both the *Walk Away* and *Follow-Flee* strategies. At the high population density values, neither strategy is able to produce substantial levels of cooperation. At low population density values, both strategies can practically guarantee the complete adoption of cooperation. However, it is clear that the *Follow-Flee* mobility strategy capitalizes on the grid dilution much earlier, and to a slightly greater extent than the *Walk Away* strategy. *Follow-Flee* is capable of promoting the

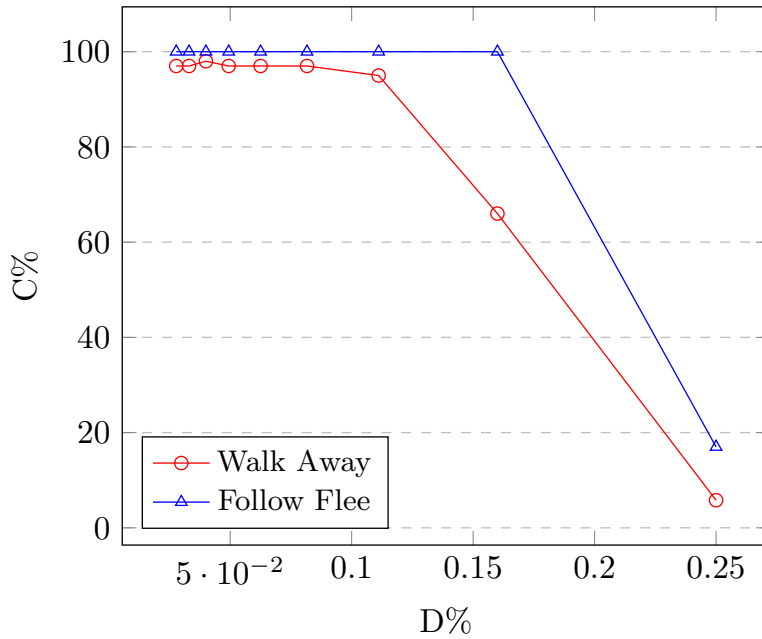


Figure 3.5: Average percentage of cooperator victories,  $C$ , for both the *Walk Away* and *Follow-Flee* mobility strategies, as a function of density,  $D$ .

evolution of cooperation in a greater percentage of simulations using harsher environments.

While it is true that the *Walk Away* mobility strategy does achieve favourable results in low densities, it has already been shown (Vainstein and Arenzon, 2014) that cooperation is enhanced by highly mobile agents in these environments. Density has such a significant influence on the emergence of cooperation due to the fact that it directly impacts the number of interactions cooperative agents may have with defectors, and determines the space in which agents may avoid such unfavourable interactions.

#### 3.2.4 Directly Comparing both Mobility Strategies

In this experiment, both strategies are used in each simulation in order to conduct a direct comparison. The *Naïve* strategy is removed as an option for players to keep the strategy proportions and population size constant, and to remove any additional complexities the presence of a third strategy may poten-

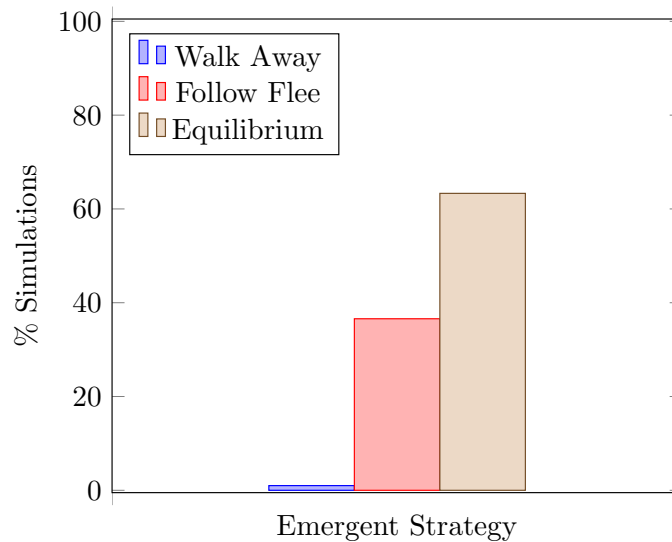


Figure 3.6: Average percentage of simulations resulting in cooperators adopting either *Follow-Flee*, *Walk Away*, or a mix of both when in direct competition, using  $L = 25$ ,  $r = 25$ , and  $s = 15$ .

tially introduce. The baseline environmental and evolutionary settings outlined in Table 3.3 are restored. As both *Walk Away* and *Follow-Flee* are both mutually cooperative, it is not expected that an evolutionary bias would favour either strategy once the defectors died out. The percentage of simulations where the *Walk Away* strategy becomes dominant, where *Follow-Flee* dominates, and the percentage of simulations where both strategies co-exist are recorded.

Cooperation emerges in almost 100% of simulations in which both mobility strategies compete directly. Table 3.6 shows that in 63% of simulations neither strategy becomes completely dominant with both able to coexist within the population. However, *Follow-Flee* emerges as the dominant mobility strategy for cooperators in 36% of the simulations, and the *Walk Away* strategy only emerges in <1% of simulations. Furthermore, as shown in Table 3.7, of the simulations resulting in an equilibrium of mobility strategies, the ratio of strategies is heavily skewed toward *Follow-Flee*. In fact, it may be that *Follow-Flee* mobility strategy is invading *Walk Away*, despite the fact that there should be no selective bias between two mutually cooperative strategies.

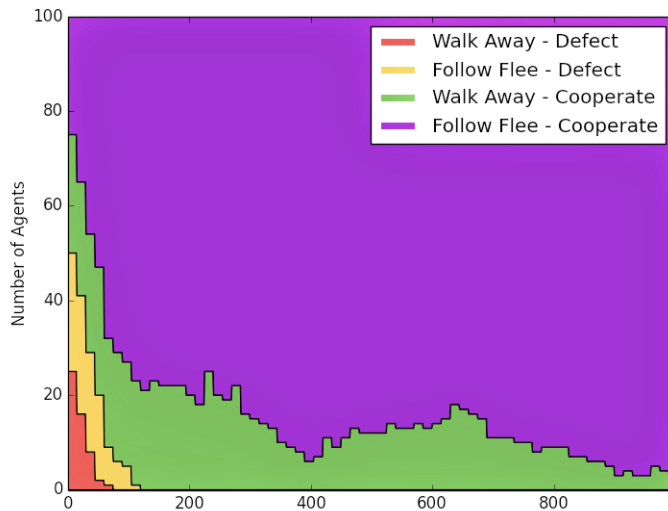


Figure 3.7: The number of agents of each strategy over the course of single, though typical, simulation, using both the *Walk Away* and *Follow-Flee* mobility strategies,  $L = 25$ ,  $r = 25$ , and  $s = 15$ , which resulted in the evolution of cooperation.

### 3.3 DISCUSSION AND CONCLUSION

In summary, *Follow-Flee*, a contingent mobility strategy for playing the spatial PD was presented, the results of experiments designed to compare it to the noted *Walk Away* strategy were described, and in doing so its superiority in promoting the evolution of cooperation was demonstrated. Both strategies were first independently tested and compared using a population of agents in a variety of evolutionary environments, including various density and reproductive settings, and then competed head-to-head in a single set of simulations. In every experiment conducted, the *Walk Away* mobility strategy was outperformed by the *Follow-Flee* strategy by significant margins. It was demonstrated that *Follow-Flee* is more resistant to the invasion of defectors, that it produces a greater percentage of cooperators victories in a wider range of evolutionary settings, and that it is more successful in higher density environments. Furthermore, it was demonstrated that the *Follow-Flee* contingent mobility strategy can invade *Walk Away* despite the fact that both are mutually cooperative strategies.

The performance of *Walk Away* as demonstrated in Aktipis' paper (2004) was not replicated in this model. In that work, the traditional restriction of one agent per cell is relaxed, and the interaction radius of agents is reduced to those in the same cell. In addition, agents only participate in one 2-player game per turn, ignoring and oft-times excluding other agents from interactions. These incongruous environmental features, in combination with rules of the *Walk Away* strategy results in mutually cooperative pairings being unexpectedly difficult to break up or be exploited by defectors, giving cooperators a built-in advantage. The high levels of cooperation reported in this work may instead be credited to the environment implementation rather than solely to the *Walk Away* strategy itself.

The success of the *Follow-Flee* contingent mobility strategy is attributed to its highly mobile, proactive nature. As illustrated in the first experiment, *Follow-Flee* is capable of inducing the emergence of cooperation in a far greater percentage of simulations. The cooperators using *Follow-Flee* are capable of increasing their number of mutually cooperative relationships by following cooperators, thus maintaining a higher average payoff, thus giving them an evolutionary edge. In contrast, clusters of *Walk Away* cooperators are immobile, which prevents them from actively seeking out new mutually cooperative interactions. The 'flee' action, common to both strategies, is crucial for agents to avoid repeated interactions with defectors. However, the *Walk Away* cooperators cannot knowingly maintain these beneficial relationships when being pursued by defectors, and so can more easily be invaded. Results indicate that the *Follow-Flee* strategy can invade *Walk Away*, even though both strategies always cooperate, due to the higher payoffs accumulated due to the formation of larger cooperative clusters that can evade defectors.

The strengths of *Follow-Flee* lie in its flexibility and simplicity. Previously, it has been stated that cooperation is enhanced in the presence of mobility (Vainstein et al., 2007; Yang et al., 2010; Meloni et al., 2009), but only when those mobility rates were low or moderate. However, high levels of cooperation were

generated in this model's highly dynamic environment using the *Follow-Flee* strategy, which only relies on limited, local information. A promising contingent mobility strategy has been presented that is extremely successful at spreading cooperation throughout a mobile population without complex computation, costly memories, or central control.



## EVOLVING CONTINGENT MOBILITY STRATEGIES FOR THE 2-PLAYER AND N-PLAYER PRISONER'S DILEMMA

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This chapter presents work investigating the co-evolution of cooperation and mobility in the spatial Prisoner's Dilemma. Agent interactions are defined using both the classical 2-Player game and a generalised N-Player PD. The extension to the game is chosen because it has not been widely studied in this domain and it represents a more difficult problem for contingent mobility to solve. An evolutionary framework is adopted, in which agent mobility strategies are represented by a genotype to capture a range of movement actions.

The interaction models are compared in terms of their ability to promote the evolution of cooperation. Additionally, the effect of varying the environment density is examined, and the mobility strategies evolved in these conditions are analysed with respect to both cooperative and non-cooperative outcomes. Finally, a set of metrics is defined to both qualitatively and quantitatively evaluate the formation of cooperative clusters.

### 4.1 METHODOLOGY

In this chapter, much of the same methodology as described in Chapter 3 is used with some notable exceptions. The evolutionary process, and the simulation processes and structure all remain constant as described in Table 3.3 from Chapter 3. The representation of agents is expanded upon to allow for the evolution of mobility, and a second N-Player interaction model is described. In the last chapter, lower grid densities were identified as being beneficial to the evolution of cooperation. The baseline value for the grid size is increased to  $L = 30$  in order

to aid in the analysis of the mobility strategies evolved when cooperation emerges by increasing the likelihood and frequency of this outcome.

#### 4.1.1 *Agent Genotype Representation*

In this model, each agent has an 6-bit genotype, which encodes the actions it can perform in each of the following scenarios, i.e. when it meets: (1) only cooperator(s), (2) only defector(s), or (3) both cooperator(s) and defector(s). Each action is represented by a 2-bit gene, see Table 4.1.

Scenarios 1 and 2 capture the following four behaviours:

- stay still (00),
- follow a neighbour (01),
- flee from a neighbour (10),
- and move randomly (11).

Scenario 3 captures the following joint four behaviours:

- follow cooperator(s) and follow defector(s) (00),
- follow cooperator(s) and flee from defector(s) (01),
- flee from cooperator(s) and follow defector(s),
- and flee from cooperator(s) and flee from defector(s) (11).

If an agent has no neighbours it explores by moving to an adjacent free location at random. The mobility strategies select new locations for agents to move to in a manner similar to that described in Section 3.1.3.

#### 4.1.2 *N-Player Interaction Model*

A well known formulation (Boyd and Richerson, 1988) for the N-Player PD is adopted in this work. The payoff obtained by a strategy which defects given  $i$  cooperators as  $D(i)$  and the

Only C	Only D	C and D
<b>00</b> -xxxx	xx- <b>00</b> -xx	xxxx- <b>00</b>
<b>01</b> -xxxx	xx- <b>01</b> -xx	xxxx- <b>01</b>
<b>10</b> -xxxx	xx- <b>10</b> -xx	xxxx- <b>10</b>
<b>11</b> -xxxx	xx- <b>11</b> -xx	xxxx- <b>11</b>

Table 4.1: An illustration of the potential agent genotype configurations in each of three environmental scenarios.

payoff obtained by a cooperative strategy given  $i$  cooperators as  $C(i)$ . Defection represents a dominant strategy, that is, for any individual, moving from cooperation to defection is beneficial for that player in that they still receive a benefit without the cost:

$$D(i) > C(i) \quad 0 < i \leq N - 1 \quad (4.1)$$

However, if all participants adopted this dominant strategy, the resulting scenario would be a sub-optimal, and from a group point of view, irrational outcome:

$$C(N) > D(0) \quad (4.2)$$

If any player changes from defection to cooperation, the whole society performs better:

$$(i+1)C(i+1) + (N-i-1)D(i+1) > (i)C(i) + (N-i)D(i) \quad (4.3)$$

Letting  $B$  be a constant representing social benefit,  $c$  be the cost of cooperation and  $i$  the number of cooperators from a group of  $N$  agents, the following payoffs are used:

$$C(i) = \frac{B \times i}{N} - c \quad D(i) = \frac{B \times i}{N} \quad (4.4)$$

The following constraints hold:  $B > c$  and both  $B$  and  $c$  are positive values.

Considering the  $N$ -Player dilemma, when  $N = 2$  and attempting to align with the classical interpretation of the 2-Player PD,

it is also required that  $B < 2c$ . The values chosen in this research that are in keeping with previous studies in the field are:  $B = 5, c = 3$ .

For example, in mapping this back to the two player games, the payoff matrix as shown in Table 4.2 is used. Work done by Hauert et al. (2006) also suggest that a unifying approach is correct when comparing different social dilemma games.

	C	D
C	2, 2	- 0.5, 2.5
D	2.5, - 0.5	0, 0

Table 4.2: Values for the Game Matrix of the N-Player Prisoner's Dilemma game with  $N = 2$ .

In simulations using the N-Player interaction model, an agent participates in the game with all of its immediate neighbours simultaneously; the number of such neighbours determines the number of participants in the game. In the 2-Player interaction model, an agent participates in individual 2-Player games with each of its immediate neighbours separately.

#### 4.1.3 Cluster Metrics

The aim of this work is to definitively determine that the clustering of cooperators is the primary cause for the proliferation of cooperation throughout a population. It is important to establish a metric for clusters as the concept is oftentimes ill-defined. Much previous research asserts the existence and formation of clusters without explicit measures of the number and type of clusters formed. In a recent review of methods to bolster network reciprocity (Kabir et al., 2018), two factors of significant influence were identified: the shape of the clusters, and the ability to expand 'perfect' cooperator clusters.

In this work, a cooperative cluster,  $c$ , is defined as a set, with cardinality greater than one, of spatially contiguous cooperative agents. Three cluster metrics are also defined to analyse the formation of clusters over the course of a simulation. Dur-

ing each simulation, a snapshot of the population is taken every 5 timesteps to calculate and record the cluster metrics. These are:

1. the number of clusters in the population,
2. the average size of clusters,
3. the cluster quality.

Cluster quality is calculated by first counting the number of neighbours of each agent in a cluster, calculating the average neighbourhood size for that cluster, and then calculating the average across all clusters in the population. This metric gives an indication of how tightly packed the agents are their cluster.

#### 4.2 SIMULATION RESULTS

In this chapter, the same experimental set up is used as described in Chapter 3. In the first experiment, two sets of similar simulations are investigated, using both the 2-Player and N-Player interaction models, comparing their respective abilities to promote the evolution of cooperation. The interaction models are also compared in terms of the evolved mobility strategies they produce. Additionally, the interaction models are analysed using a number of metrics to both quantitatively and qualitatively measure the cooperative clusters formed. The second set of experiments compare the relative success of both interaction models at different density levels (varying  $L$ ), and investigate the influence of density, if any, on the formation of cooperator clusters. In the final experiment, the evolved mobility strategies produced by both interaction models are seeded into the initial population. These evolved strategies are evaluated in terms of their ability to promote the evolution of cooperation in a range of environment densities, and in terms of their ability to produce cooperator clusters.

The mobility strategies of agents are randomly initialized, and the game strategies (to cooperate or defect) are assigned

Interaction Model	Cooperator Victories
2-Player	33.2%
N-Player	25.8%

Table 4.3: Percentage of simulations resulting in the evolution of cooperation for the 2-Player and N-Player interaction models, using  $L = 30$ ,  $r = 20$ , and  $s = 25$ .

in equal proportion; both are subject to evolution. In one set of simulations, the population interacts using the 2-Player interaction model, while the other set uses the N-Player model.

#### 4.2.1 *Comparing 2-Player and N-Player Mobility*

In this experiment, environmental and evolutionary settings favourable to the evolution of cooperation, identified in the previous chapter, are used to compare and contrast the game models. The generation length  $s = 25$  and replacement rate  $r = 20$ , which constitute the lifespan of agents, are fixed, using population density of  $L = 30$ .

In Table 4.3, it is shown that the 2-Player interaction model is more effective at promoting the evolution of cooperation, using these settings, than the N-Player model. Cooperation emerges in simulations using the 2-Player interaction model roughly 33% of the time, whereas cooperation emerges in just over 25% of simulations using the N-Player interaction model. This result is in line with the literature which indicates that the N-Player PD is a more difficult game in which to promote cooperation. Additionally, simulations using the 2-Player interaction model tend to converge more quickly with less variance compared to those using the N-Player model. Simulations resulting in the emergence of defection exhibit a faster convergence speed with less variability regardless of the interaction model used.

##### 4.2.1.1 *Evolved Mobility Strategies*

At the end of every simulation, the genotypes of every agent in the population were recorded and analysed. The simulations

Evolved Behaviour	2-Player	N-Player
<b>Stay Still</b>	<b>85%</b>	<b>73%</b>
Follow	15%	27%
Flee	0%	0%
Random	0%	0%

Table 4.4: Distribution of the evolved behaviours when an agent sees only cooperators in their neighbourhood expressed in a population which evolved cooperation using both the 2-Player (left) and N-Player (right) interaction models.

were divided by the interaction model used, and whether the the simulations resulted in the emergence of cooperation or defection. In general, similar mobility strategies evolve in simulations using both the 2-Player and N-Player interaction models. However, a difference does exist between the evolved behaviours of cooperators and defectors.

Tables 4.4, 4.5 and 4.6 show the evolved mobility strategies that emerge for each of the three environmental scenarios using both the 2-Player and N-Player interaction models in simulations where cooperation is also evolved. In all scenarios, agents generally learn movement behaviours that continue cooperative interactions and, to a lesser extent, avoid interactions with defectors. Behaviours that continue defector interactions die, although at a slower rate. There is a greater selective pressure to follow cooperators than to flee from defectors.

Table 4.4 illustrates the behaviours evolved for the environmental scenario in which an agent sees only cooperators in its neighbourhood. This corresponds to the first two bits in the genotype. Agents have a strong preference to evolve to either stay still or to follow the cooperator(s); this occurs using both 2-Player and N-Player models. These two behaviours are largely similar because they both result in the agent continuing with the cooperative interaction in the next step. There is a strong selective pressure on agents to not evolve to flee or to move randomly; these behaviours would likely result in the agent discontinuing the cooperative interaction(s).

Evolved Behaviour	2-Player	N-Player
Stay	9%	1%
Follow	16%	2%
<b>Flee</b>	<b>41%</b>	<b>75%</b>
Random	34%	22%

Table 4.5: Distribution of the evolved behaviours when an agent sees only defectors in their neighbourhood expressed in a population which evolved cooperation using both the 2-Player (left) and N-Player (right) interaction models.

Evolve Behaviour	2-Player	N-Player
Follow C & D	27%	3%
<b>Follow C Flee D</b>	<b>44%</b>	<b>52%</b>
Flee C Follow D	9%	11%
Flee C & D	20%	34%

Table 4.6: Distribution of the evolved behaviours when an agent sees both cooperators and defectors, expressed in a population which evolved cooperation using both the 2-Player (left) and N-Player (right) interaction models.

Table 4.5 illustrates the behaviours evolved for the environmental scenario in which an agent sees only defectors in its neighbourhood. This corresponds to the middle two bits in the genotype. When a defector is encountered, agents have evolved to flee 40% of the time in the 2-Player game, and 75% of the time using the N-Player game. There is much more selective pressure on agents using the N-Player model in this environmental scenario to flee from defectors. This reinforces the idea that individual defectors have a much greater exploitative impact on cooperators in this version of the game. It is also worth noting that the second most commonly evolved behaviour is random, which may result in the desired action to flee from the defector.

Table 4.6 illustrates the behaviours evolved for the environmental scenario in which an agent sees both cooperators and defectors in its neighbourhood. This corresponds to the final two bits in the genotype. In this scenario agents have a strong preference to evolve behaviours that promote cooperation and

avoid exploitation. The behaviour ‘Follow C Flee D’ is most predominately evolved in both models, with the opposite action being evolved the least, and the other actions falling in between. Both interaction models evolve similar behaviours, however as in the previous scenario, this preference to flee from defectors is stronger in simulations using the N-Player model. Table 4.6 shows that contingent mobility strategies that contain the action ‘Follow C & D’ are evolved at much higher rates from populations using the 2-Player model than using the N-Player model. Additionally, the action ‘Flee C & D’ is more likely to be evolved by agents using this model.

In simulations where defection emerged, there was a lack of selective pressure on agents to produce mobility strategies. This is largely due to the defectors already having such an advantage over the cooperators in the game. In general, they learned to follow cooperators, and flee from other defectors.

#### 4.2.1.2 *Cluster Analysis*

A number of cluster metrics are developed to record and analyse the formation and growth of cooperator clusters. Over the course of a simulation, clusters are identified in the population, and measured in a variety of ways; including the number of clusters, the average size, and their quality. Figures 4.1, 4.2, and 4.3 show the outputs of these metrics over a single, though typical, simulation which resulted in the spread of cooperation using both the 2-Player and N-Player interaction models.

Each of the Figures 4.1, 4.2, and 4.3 show that the populations using the 2-Player interaction model form a greater number, larger, and high quality cooperator clusters more quickly than those using the N-Player model. In general, the clusters improve more quickly in the 2-Player model, but the clusters created by the N-Player model do eventually reach similar levels in terms of the three metrics. Variability is present in these graphs due to that fact that clusters can be divided and broken up by defectors, and by the evolutionary process itself.

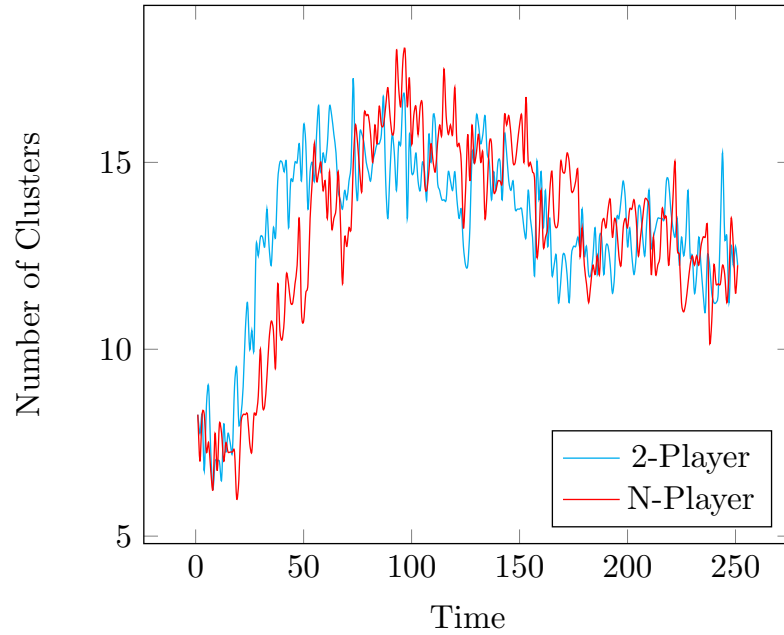


Figure 4.1: The Number of Clusters over a single, though typical, simulation in which cooperation emerges using both the 2-Player and N-Player interaction models. The measurements are taken once every 5 out of 1000 timesteps.

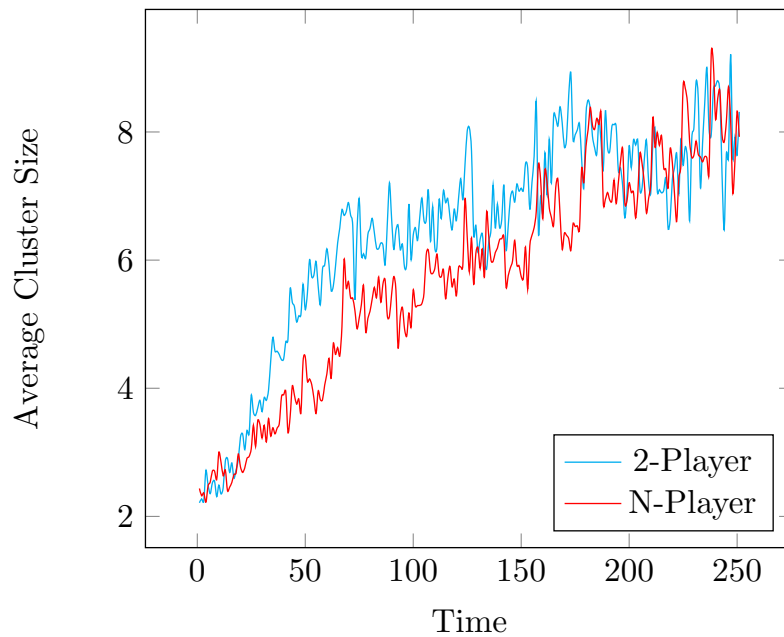


Figure 4.2: The Average Size of Clusters over a single, though typical, simulation in which cooperation emerges using both the 2-Player and N-Player interaction models. The measurements are taken once every 5 out of 1000 timesteps.

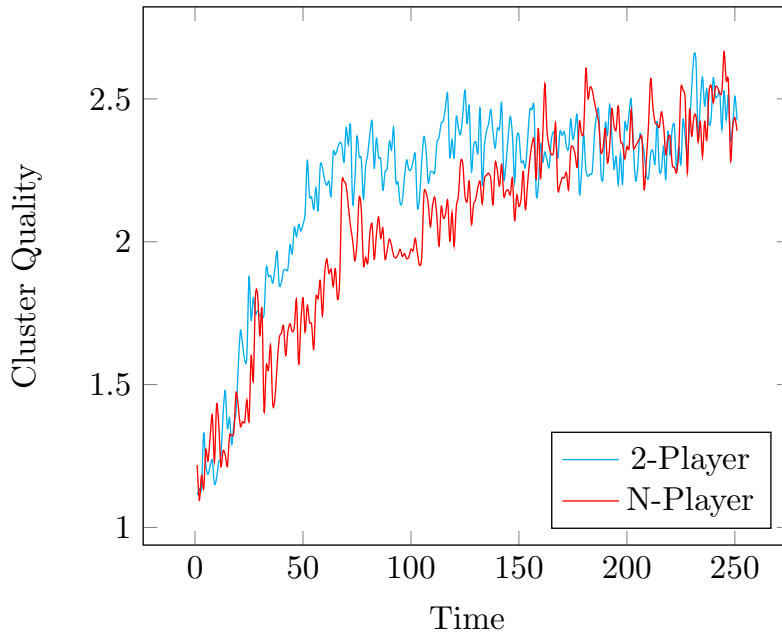


Figure 4.3: The Average Cluster Quality over a single, though typical, simulation in which cooperation emerges using both the 2-Player and N-Player interaction models. The measurements are taken once every 5 out of 1000 timesteps.

Figure 4.1 shows that the number of clusters, in both 2-Player and N-Player simulations, initially increases rapidly, then reaches a plateau, and eventually decreases. At the beginning of the simulation, the population is randomly dispersed with many small clusters forming as cooperators move and interact according to their strategy. As the defectors start to reduce in numbers, the cooperator clusters have the space and freedom to merge and grow larger, thus reducing the overall cluster count. This is reflected in Figure 4.2 where it is shown that the average cluster size, in both cases, increases over time.

Cluster quality quantifies the potential fitness of a cluster by measuring the neighbourhoods of its individuals. Figure 4.3 demonstrates that cooperator clusters improve over time using both interaction models, furthering the spread of cooperation.

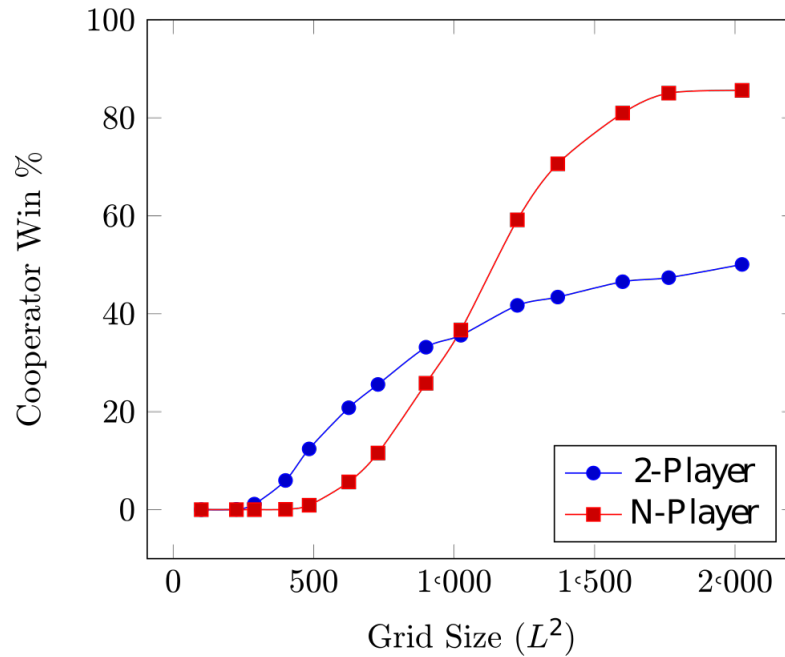


Figure 4.4: Average percentage of simulations which result in the evolution of cooperation for a range of density settings, i. e. values of  $L$ , using both the 2-Player and N-Player interaction models.

#### 4.2.2 2-Player and N-Player: Varying Density

In the previous experiments, the percentage of cooperative outcomes and the evolution of movement strategies was a function of the agent interactions. The evolutionary trajectories are influenced by the ratio of cooperative to defective interactions, which is often determined by the density of the environment in which the agents interact. In this experiment, the impact of varying the environment density is investigated. Density is defined as  $D = A/L^2$ , where  $A$  is the size of the population, and  $L$  is the length of the lattice grid. In this work, the size of the physical grid occupied by the agents is varied while keeping the population constant. This is done to ensure to minimize the impact of the evolutionary process, which also uses the population size.

Figure 4.4 shows the percentage of simulations which result in the evolution of cooperation in both the 2-Player and N-Player interaction models for a range of environment densities.

Firstly, the graphs show that for both interactions models as the grid sizes increases, thereby reducing the environment density, the percentage of simulations resulting in the evolution of cooperation also increases. Additionally, it is evident that there are some density levels for which cooperation cannot evolve. There is not enough space within the grid for agents to move freely and so defection dominates in the vast majority of simulations. These conditions echo the traditional spatial models in which agents occupy every cell and no movement is possible and defection is more likely to spread and dominate the population.

Figure 4.4 shows that the 2-Player interaction model produces higher levels of cooperation in the higher density settings. However, this model can only promote the evolution of cooperation in just above 50% of simulations regardless of the favourable density. Surprisingly, using these same settings the N-Player interaction model can promote the evolution of cooperation in a far greater percentage (80%) of simulations, given a sufficiently low population density. The N-Player model overtakes the 2-Player model in its ability to promote cooperation at a density of approximately  $D = 0.1$ , i.e. a grid size of  $32 \times 32$  (1024 cells). This demonstrates that despite the difficulty of promoting the evolution of cooperation, it may emerge in N-Player games given an appropriate environment.

#### 4.2.2.1 *The impact of Density on Cluster Formation*

Figure 4.5 and 4.6 compare the clustering produced by high and low density environments using each of the three cluster metrics for the 2-Player and N-Player interaction models respectively. In low density environments, a larger number of small, low quality cooperator clusters are created by both interaction models. Here, the population is more dispersed throughout the environment, resulting in fewer overall interactions leading to the creation of fewer clusters. On the other hand, in the high density environment, a smaller number of large, high quality clusters are created when the smaller clusters merge together. Additionally, Figures 4.5 and 4.6 illustrate the trend of the rapid

creation of cooperation clusters, followed by a more gradual decline in the number of clusters in favour of size and quality.

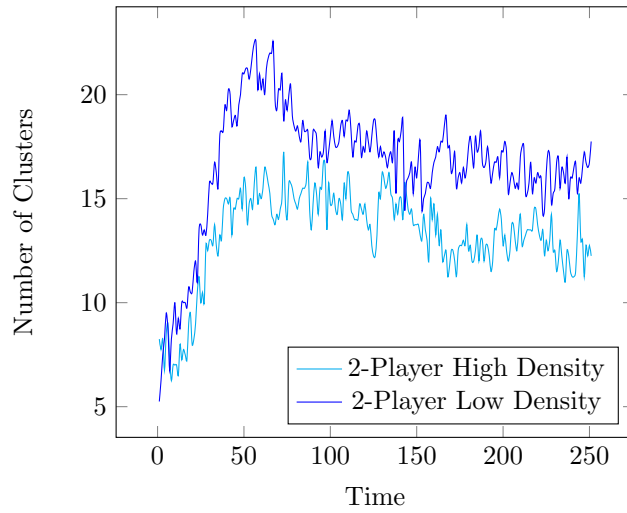
Similar to the difference between the clusters created, in the high and low density environments, the N-Player interaction model, as shown in Figure 4.6, creates a larger number of small cooperator clusters in comparison to the 2-Player model.

#### 4.2.3 *Seeding the Evolved Mobility Strategies*

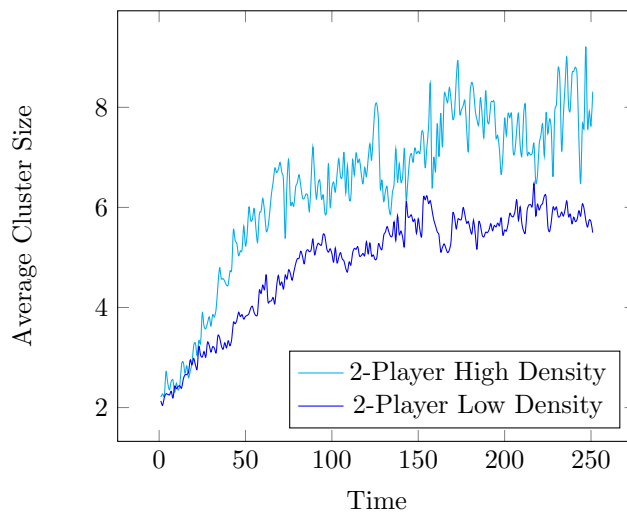
Previously, the mobility strategies of agents were randomly assigned, and it took several generations for ‘good’ strategies to emerge. In this final experiment, the initial population of each simulation is seeded with these evolved mobility strategies in order to test their effectiveness at promoting the evolution of cooperation. Cooperators are seeded with the genotype  $\{0,0,1,0,0,1\}$ , which means that they will follow cooperators and flee from defectors, unless they only see other cooperators in which case they stay still. Defectors are given the genotype  $\{0,1,1,0,0,1\}$ , which translates to ‘follow cooperators and flee from defectors’.

As before, the strategies are seeded into populations using both the 2-Player and N-Player interaction models, and the agents are assigned to cooperate or defect in equal proportion. Additionally, the impact of the variation in density on these seeded behaviours, using both interaction models, is also considered. If these ‘good’ strategies can help cooperators to form cooperative clusters, then higher levels of cooperation are expected across the various environmental densities.

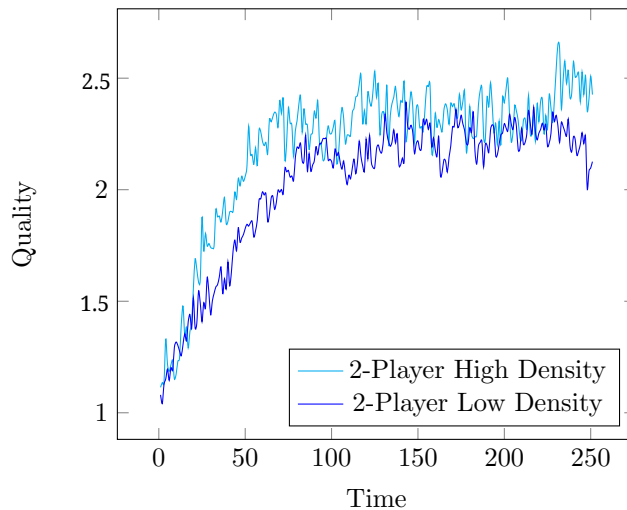
Figure 4.7 shows the percentage of simulations using seeded mobility strategies which result in the evolution of cooperation using both interaction models for a range of environment densities. In both sets of simulations, the seeded mobility strategies are able to promote the evolution of cooperation to a great degree of success for a wider range of environment densities. These are far greater levels of cooperation than that achieved using unseeded strategies, which were randomly initialized,



(a) Number of Clusters

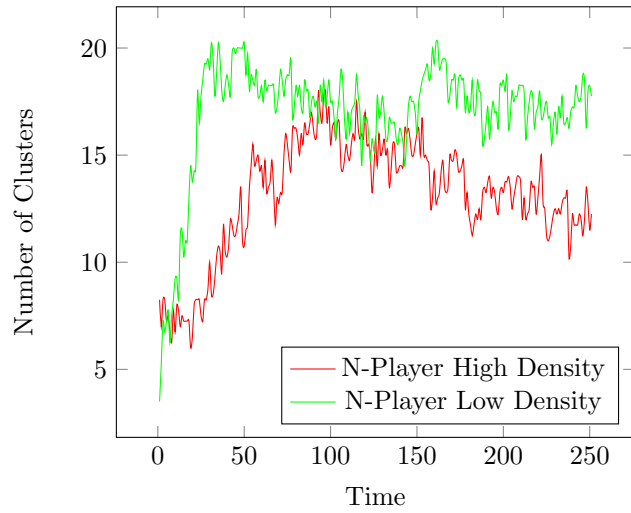


(b) Average Cluster Size

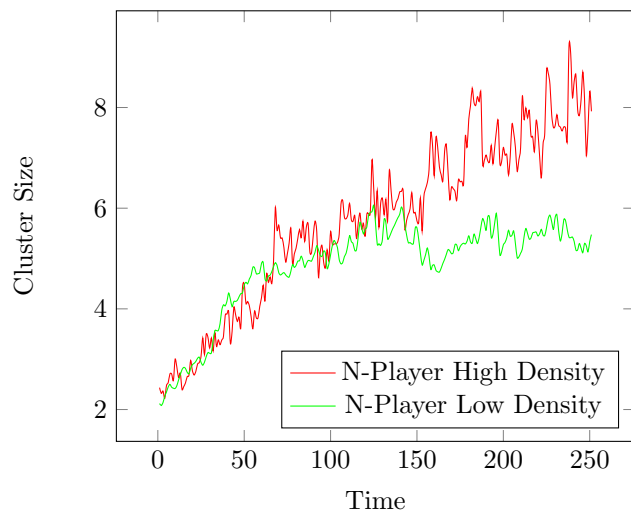


(c) Average Quality

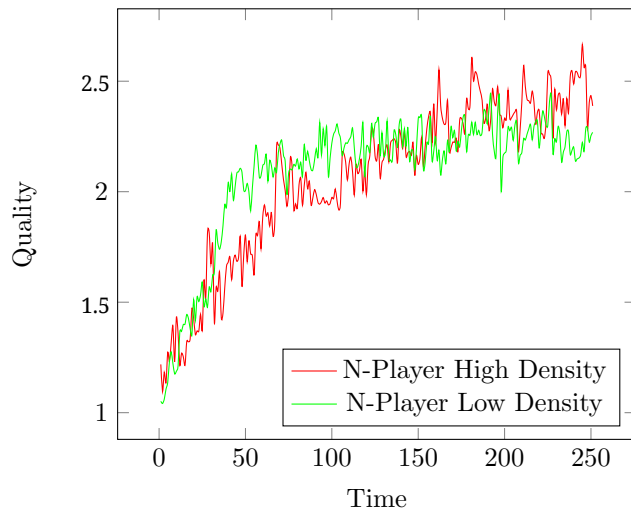
Figure 4.5: The Number of Clusters (a), Average Cluster Size (b), and Average Cluster Quality (c) over a single, though typical, simulation in which cooperation emerges using the 2-Player interaction models at two density levels. The measurements are taken once every 5 out of 1000 timesteps.



(a) Number of Clusters



(b) Average Cluster Size



(c) Average Quality

Figure 4.6: The Number of Clusters (a), Average Cluster Size (b), and Average Cluster Quality (c) over a single, though typical, simulation in which cooperation emerges using the N-Player interaction models at two density levels. The measurements are taken once every 5 out of 1000 timesteps.

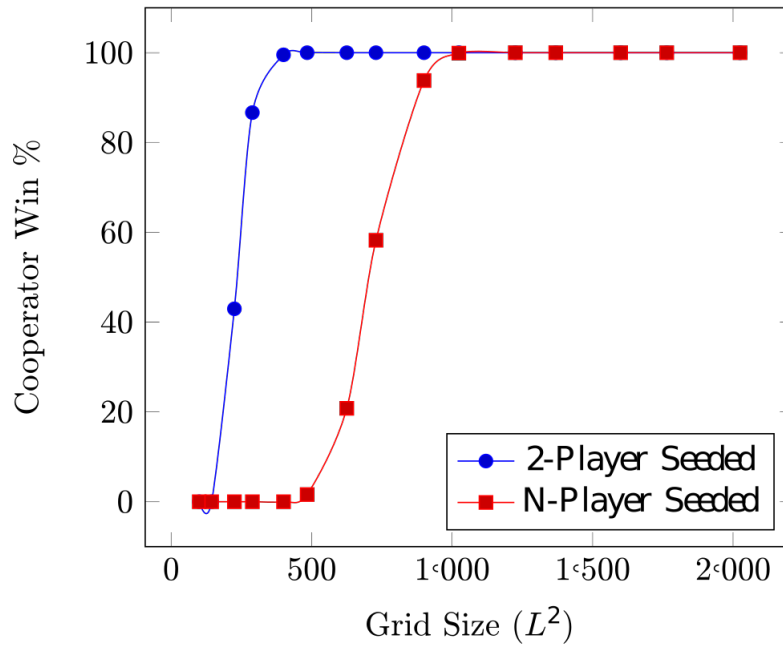


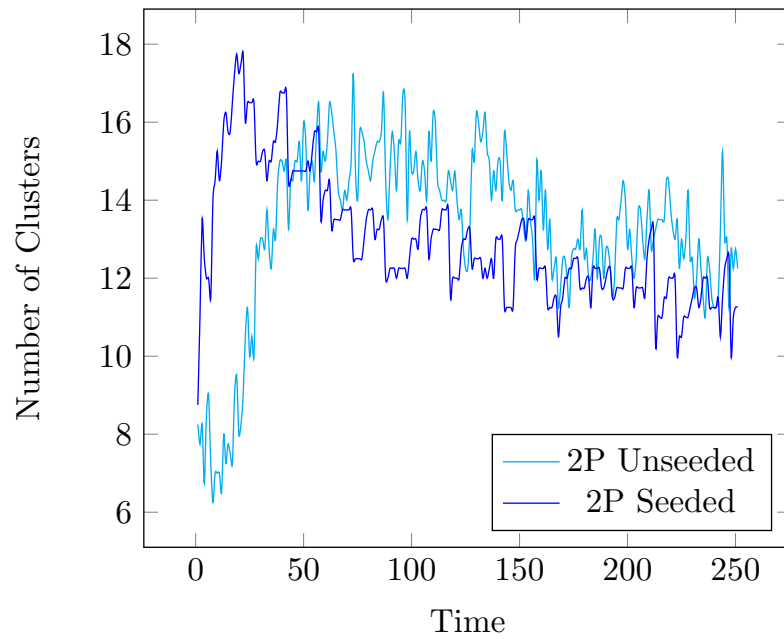
Figure 4.7: Average percentage of simulations using seeded mobility strategies which result in the evolution of cooperation for a range of density settings, i. e. values of  $L$ , using both the 2-Player and N-Player interaction models.

reaching a 100% success rate for both the 2-Player and N-Player interaction models. However, the 2-Player model does perform better than the N-Player model in promoting cooperation in the high density environments. Agents using the N-Player model are more hindered by the exploitative nature of defectors, who are also using an seeded mobility strategy.

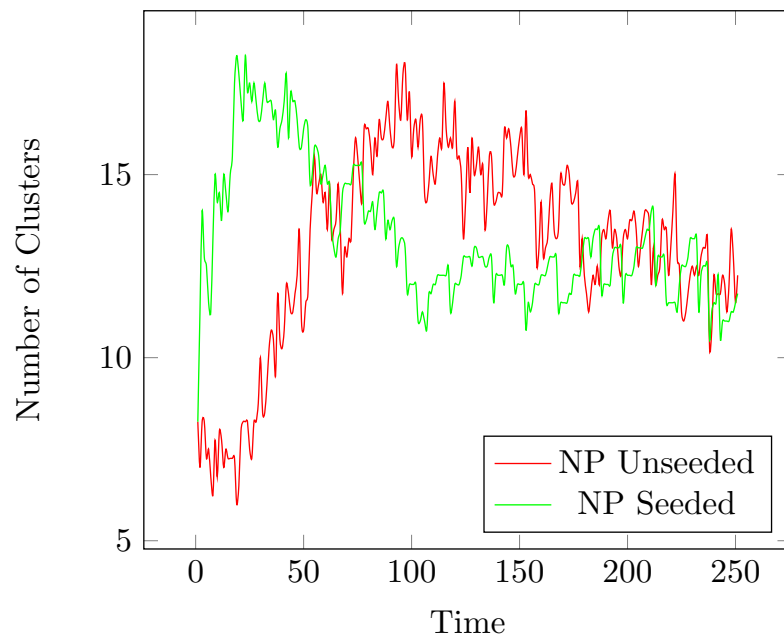
#### 4.2.3.1 Cluster Analysis with Seeded Strategies

Figure 4.8 compares the clustering produced by seeded and unseeded populations using the 2-Player and N-Player interaction models respectively in simulations that resulted in the evolution of cooperation. The same environmental density and evolutionary settings as described in the first experiment are used.

In both scenarios, the seeded strategies generate a significantly greater number of clusters in the early generations compared to the evolving strategies in the unseeded populations. Additionally, the size of the clusters is shown to be more rapidly



(a) 2-Player



(b) N-Player

Figure 4.8: The Number of Clusters over a single, though typical, simulation in which cooperation emerges from a population using seeded and unseeded mobility strategies, using the 2-Player (a) and N-Player (b) interaction models. The measurements are taken once every 5 out of 1000 timesteps.

increasing in the seeded populations as the number of clusters decreases. Once the unseeded population evolves 'good' mobility strategies a similar pattern for the number of clusters emerges. These patterns are also observed when using the other two cluster metrics.

### 4.3 DISCUSSION

The co-evolution of cooperation and contingent mobility strategies has been demonstrated using both the 2-Player and N-Player PD in a range of environment densities. Additionally, a detailed analysis of simulations in which cooperation evolves reveals that contingent mobility strategies promote the creation of cooperator clusters which in turn promotes the evolution of cooperation.

Traditionally, it has been difficult to promote the evolution cooperation using the N-Player PD. However, the addition of mobility capabilities does support the emergence of cooperation in this game. As expected, the 2-Player interaction model was more successful at promoting cooperation in populations with higher densities using an unseeded population. In these environments while the chances of encountering a defector are higher, they have less of an exploitative impact on individual or clusters of cooperators. However, the N-Player model surprisingly outperformed the 2-Player model in the lower densities in terms of the vast differences in the levels of evolved cooperation. It may be the case that cooperator clusters using the N-Player model, when unhindered by defectors, generate higher payoffs than their 2-Player counterparts. This may also be due to the differences between the mobility strategies evolved by the two interaction models.

Generally, the evolved mobility strategies produced by the 2-Player and N-Player interaction models were largely similar. However, agents using the N-Player model were under significantly more selective pressure to evolve to avoid repeated interactions with defectors. In these settings, agents were more

likely to choose behaviours that would result in them fleeing from cooperators than following defectors. As a result, it becomes more difficult for large cooperators to grow and persist in populations using the N-Player interaction model. This is reflected in the cluster analysis where this model tended to create a larger number of small clusters.

The population did not always evolve to a single best mobility strategy; random fluctuation and lack of relevant stimuli resulted in simulations in which agents converged on several strategies that were genotypically different, but phenotypically similar. It is true that 'stay still' emerged as the most common behaviour for reacting to cooperators regardless of the interaction model used. However, when the defectors died out during a simulation the selective pressure to avoid interacting with them also died out. Furthermore, adopting a random movement can often have the same effect as fleeing from or indeed following an agent.

In forming these clusters, cooperators can increase their number of mutually cooperative interactions, thereby boosting their score. However, these cooperative clusters can be exploited by defectors unless they employ strategies that can avoid repeated exploitative encounters. High levels of cooperation are observed coupled with evolved movement strategies that encourage the formation of these larger self-preserving clusters free from the influence of defectors. Cooperation emerges when cooperators learn mobility strategies that allow them create these dynamic clusters. Evidence for the clustering of these mobility strategies is observed in simulations that resulted in widespread cooperation for all cluster metrics considered (the number of clusters, the cluster size, and the cluster quality).

#### 4.4 CONCLUSION

In this chapter, a spatial model was presented, in which agents co-evolve their movements with their strategy for playing the PD. Two versions of this game were considered; the traditional

2-Player game and an extended N-Player version of the game which allows for multiple agents to interact simultaneously. The two interaction models are discussed in terms of their ability to promote the evolution of cooperation, the mobility strategies they produce, and their influence on the formation of cooperative clusters. The environment density was also varied in a systematic manner and its effect on the evolution of cooperation using both interaction models was discussed.

A number of cluster metrics were defined to analyse the clustering of cooperators in this strict spatial environment. These include the number of clusters in a population, the average size of those clusters, and their quality.

Through experimentation, it was shown that contingent mobility strategies help promote the evolution of cooperation in the N-Player PD, for a range of environment densities, by facilitating the growth of cooperator clusters. Furthermore, it is shown that seeding a population with these evolved mobility strategies can almost guarantee the evolution of cooperation in both the 2-Player and the N-Player interaction models. These mobile strategies are adept at spreading cooperation throughout a mobile population, by forming cooperative clusters, without the need for complex computation, or costly memories.



## EVOLVING A GENERALISED CONTINGENT MOBILITY STRATEGY

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This chapter presents work further investigating the co-evolution of cooperation and mobility in the spatial Prisoner's Dilemma. A more expressive mobility model is proposed, in which the agent genotype is extended to encode a wider range of movement behaviours. Additionally, a more thorough investigation into the influence of various environmental and evolutionary factors is presented. Specifically, the influence of population density and agent lifespan (outlined in Chapter 3), the interaction model of the game (outlined in Chapter 4), and the placement strategy of new agents are investigated on the co-evolution of cooperation and contingent mobility strategies.

### 5.1 METHODOLOGY

In this chapter, much of the same methodology as described in Chapter 3 and 4, is utilised with some notable exceptions. The physical environment, the evolutionary process, the two interaction models, and the simulation processes and structure remain constant as described in Table 3.3. The representation of agents is expanded upon to allow for a generalised and more expressive model for the evolution of mobility. Furthermore, a second placement mechanism in the evolutionary process is introduced, whereby new agents are placed in the neighbourhood of their parent. As in Chapter 4, agents interact using two versions of the PD; the standard 2-Player interaction model and the N-Player model are both considered. Finally, the baseline value for the grid size is again increased to  $L = 40$  in order to aid in the analysis of the evolved mobility strategies using the more expressive yet difficult evolutionary landscape.

Only C	Only D	C with D	D with C
00 -xxxxxx	xx- 00 -xxxx	xxxx- 00 -xx	xxxxxx- 00
01 -xxxxxx	xx- 01 -xxxx	xxxx- 01 -xx	xxxxxx- 01
10 -xxxxxx	xx- 10 -xxxx	xxxx- 10 -xx	xxxxxx- 10
11 -xxxxxx	xx- 11 -xxxx	xxxx- 11 -xx	xxxxxx- 11

Table 5.1: An illustration of the potential agent genotype configurations in each of four environmental scenarios.

### 5.1.1 Extended Mobility Model

Each agent has an 8-bit genotype, which encodes the actions it can perform. Each action is represented by a 2-bit gene capturing the following four behaviours: remain where they are i.e. stay still (00), follow a neighbour (01), flee from a neighbour (10), and move randomly (11). Given the bit positions from left to right, the agents will perform one of these actions in each of the following scenarios, see Table 5.1, i.e. when it meets:

- only cooperator(s) (bit position 0 and 1);
- only defector(s) (bit position 2 and 3);
- cooperator(s) in a neighbourhood with defector(s) present (bit position 4 and 5);
- defector(s) in neighbourhoods with cooperator(s) present (bit position 6 and 7).

If an agent has no neighbours it explores by moving to an adjacent free location at random.

An agent's environmental scenario determines how each location in its neighbourhood is evaluated, which is determined by its genotype. Each location is assigned a score based on its proximity to neighbouring agents and the game strategy of those neighbours, similar to the methodology presented in Chapter 3. In scoring a neighbourhood containing multiple agents, the scores for each location are first calculated individually per agent, and then accumulated into a combined score. This in

C	1	0	1	1	1	C	2	1
1	X (1)	0	0	X (0)	0	1	X (1)	0
0	0	0	0	D	0	0	D	0

Table 5.2: An illustration of the calculation used to determine the move performed by an agent with the genotype  $\{0,1,1,0,0,1,1,0\}$  where: all neighbours are cooperators (left); all neighbours are defectors (middle) and neighbours are both cooperators and defectors (right).

turn is used to rank those available locations. In this way, if the gene section reads:

- ‘stay still’ (00), the current cell location scores zero, while the other cell locations subtract one;
- ‘follow’ (01), the cell locations adjacent to each neighbour are incremented by one, while the other cell locations score zero;
- ‘flee’ (10), the cell locations adjacent to each neighbour score zero, while the other cell locations are incremented by one;
- ‘random’ (11), all cell locations are incremented randomly by one, zero, or minus one;

based on the total score obtained for each cell, the agent moves to the highest ranking location and ties are broken by choosing a tied location at random.

For example, given an agent **X** with the genotype  $\{0,1,1,0,0,1,1,0\}$ , which translates to ‘follow cooperators and flee from defectors’. Table 5.2 outlines the results of agent **X**’s movement locations being scored in each of the non-trivial scenarios. This produces the same result as the example given in Chapter 3.

### 5.1.2 Placement Mechanism

Recently, some research (Chen et al., 2019) has focused on birth-death update rules and highlight their impact on the evolution of cooperation in structured populations. In this work, two

mechanisms are considered for placing agent offspring in the environment. The first, ‘random’ placement, chooses a location at random from the available free spaces on the grid. Similar methodologies are present in other works (Aktipis, 2004; Burgess et al., 2017). The second placement mechanism investigated places offspring in the neighbourhood of their parent. In the case where the parent’s neighbourhood is full, they are randomly placed on the grid instead. This ‘nearby’ placement approach should further strengthen the structure of clusters created between generations.

## 5.2 SIMULATION RESULTS

In this section, the experimental results of the simulations of the PD game on a diluted toroidal lattice grid in both the 2-Player and N-Player interaction model are presented. The first set of experiments comprise variations in the parameters  $r$  and  $s$  using the standard 2-Player model as described in Chapter 3; the second set of experiments comprise variations in the parameters  $r$  and  $s$  at different density levels (varying  $L$ ); the third set of experiments compare results across the two replacement strategies (‘random’ or ‘nearby’), the fourth section presents an analysis of the genotypes of agents in a specific evolutionary setting with the ‘random’ replacement strategy, and the last section investigates the N-Player PD using these environmental and evolutionary settings.

### 5.2.1 *Varying the Evolutionary Settings*

To begin, the scenarios in which a population of agents playing the PD game evolve mobility strategies to promote the evolution of cooperation are investigated by identifying optimal evolutionary settings. The generation length  $s$  and replacement rate  $r$ , which constitute the lifespan of agents, are both varied from the values 5 to 35, while the population density is fixed at  $L = 40$ . The ‘random’ placement strategy is used.

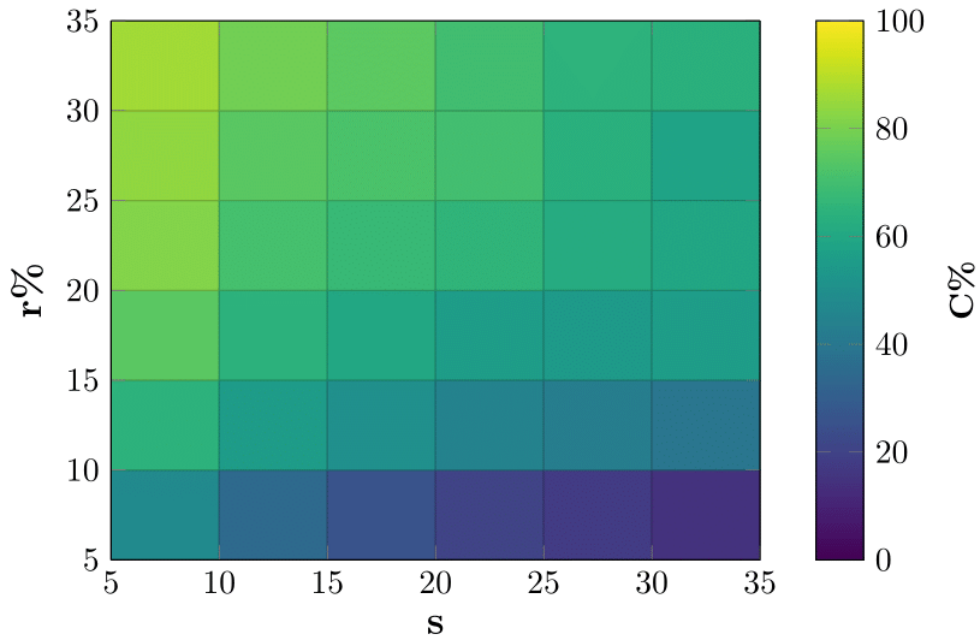


Figure 5.1: Average percentage of cooperators victories for grid size  $L = 40$  as a function of replacement rate  $r$ , and the steps per generation  $s$ .

Figure 5.1 shows the percentage of simulations that end with total adoption of cooperation, i. e. cooperator victories. Simulations always converge on either total cooperation or defection, ‘draws’ are very rare, and only occur when the convergence for a particular evolutionary setting is slow. The settings that lead to the most cooperative outcomes, on average, are high replacement rates,  $r$ , coupled with low generation lengths,  $s$ . Using these settings, cooperators dominate the population in 93% of randomly initialized simulations. It is noteworthy that the value of  $r$  has a bigger influence on the emergence of cooperation than  $s$ . In practice, this indicates that cooperation emerges more readily when fewer agents are replaced per generation, than when agents have longer to interact during a single generation and potentially be exploited by ‘free riders’ (Enquist and Leimar, 1993). However, the best results are achieved when these scenarios are combined. This suggests that the replacement process should be tuned in order for cooperation to emerge with the greatest probability.

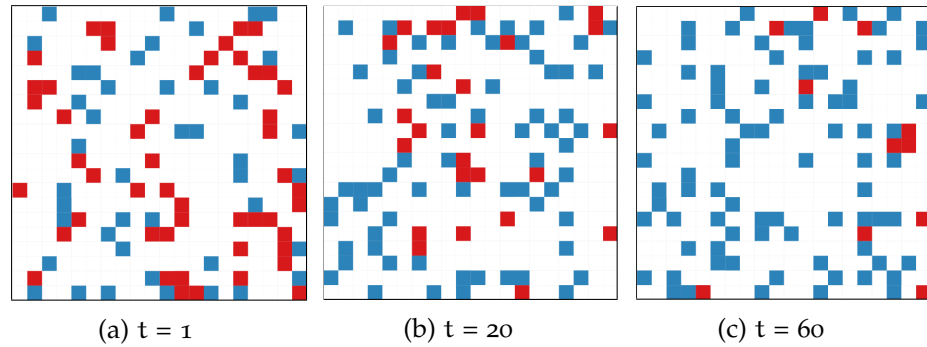


Figure 5.2: Typical distributions of agents, cooperators (blue) and defectors (red), at various timesteps ( $t$ ) in a single simulation, using  $r = 35$  and  $s = 5$ , on a  $L = 20$  diluted lattice grid. Screenshots generated using Evolpex (Cardinot et al., 2018).

Figure 5.2, shows a snapshot sequence of a population during a single, though typical, simulation that resulted in the spread of cooperation. A decline in the number of defectors over time is observed as cooperator clusters form and expand.

### 5.2.2 Varying Environment Density

The influence of density on the evolution of cooperation is investigated by repeating the previous experiment across a range of grid sizes:  $L = 25$  to  $L = 55$  in intervals of 5.

In Figure 5.3, we observe the percentages of cooperator victories across a range of both evolutionary settings and density levels. In the high density graphs, Figure 5.3 (a), we observe that extremely low levels of cooperation emerge despite the variation in evolutionary settings. These results are unsurprising as this environment is close to being fully connected. In this setting, we would expect defectors to easily invade cooperators as described by the ‘free-rider’ effect.

Figure 5.3 (b) - (d) shows an increase in the level of cooperation emerging. We observe the evolutionary settings’ growing effect on the emergence of cooperation. It becomes clear that as the population density decreases, the percentage of simulations resulting in a cooperative outcome increases, as agents have the space to learn and deploy their movement strategies.

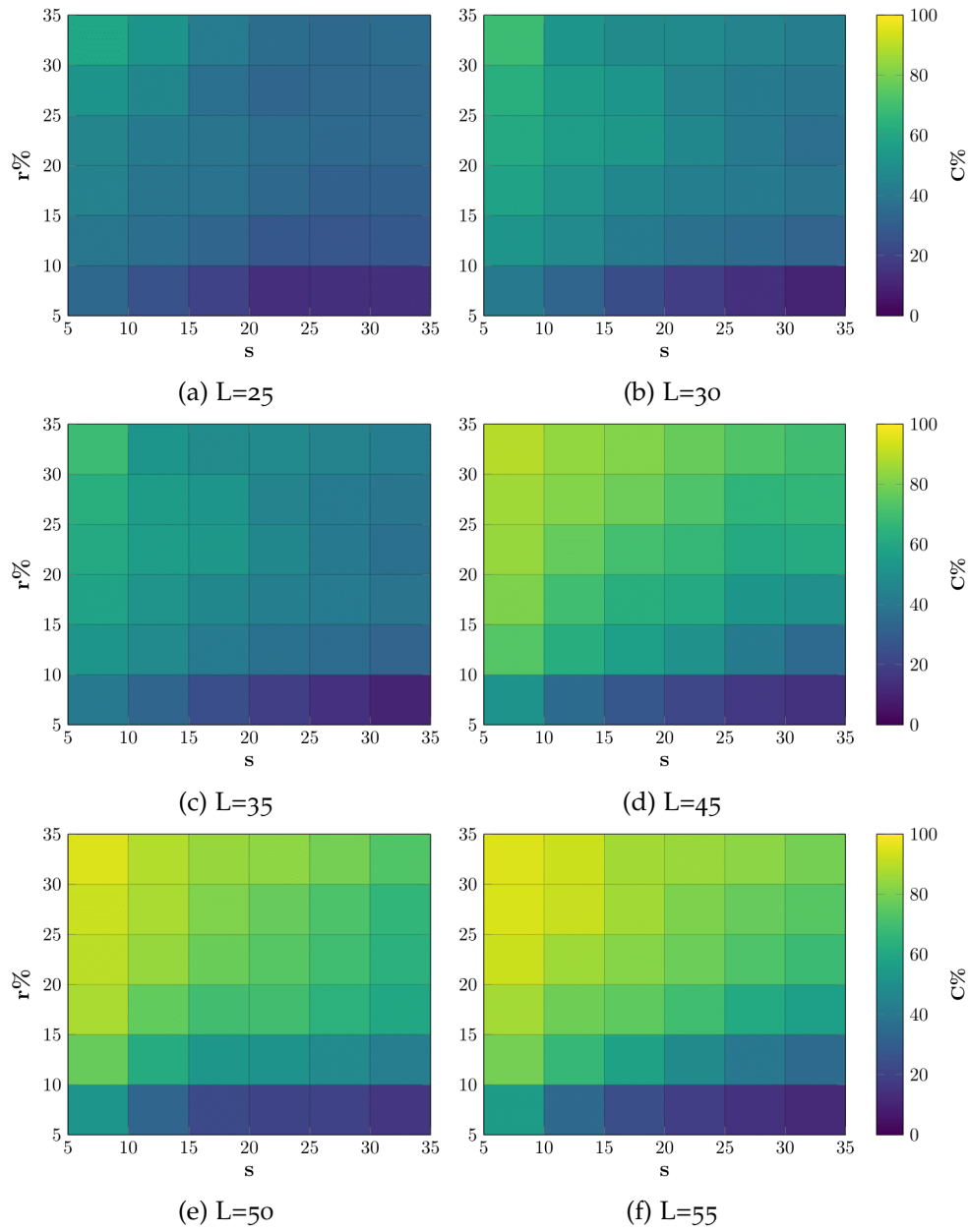


Figure 5.3: Average percentage of cooperator victories,  $C$ , for a variety of grid sizes  $L$ , as a function of replacement rate  $r$ , and the number of steps per generation  $s$ .

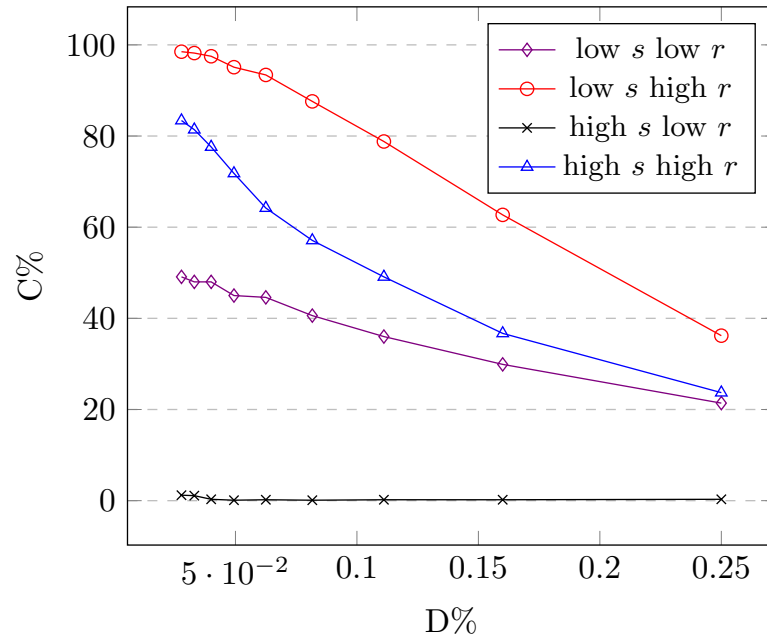


Figure 5.4: Average percentage of cooperator victories,  $C$ , for a number of evolutionary settings as a function of density,  $D$ . The low and high values are  $r = 5, 35$  and  $s = 5, 35$ .

In the low density environments, i.e. Figure 5.3 (e) - (f), the trend becomes most pronounced. We see that cooperation is able to emerge in almost 100% of simulations for a wide range of evolutionary settings. In randomly initialized simulations, cooperators have enough time and space to learn the movement strategies capable of dominating the defectors.

Figure 5.4 more clearly demonstrates the impact of density on the evolution of cooperation. In the graph, the most extreme values for the evolutionary settings,  $r$  and  $s$ , are directly observed across the density values, from very low to very high. Again, cooperation emerges most readily for low values of  $r$  and high values of  $s$ . However, for certain evolutionary settings, i. e. high  $r$  and low  $s$ , it is nearly impossible for cooperation to emerge, regardless of any variation in density.

### 5.2.3 Comparing Placement Mechanisms

In this section, the central experiment is again repeated, co-evolving agent strategy and movement pattern, using the ‘nearby’

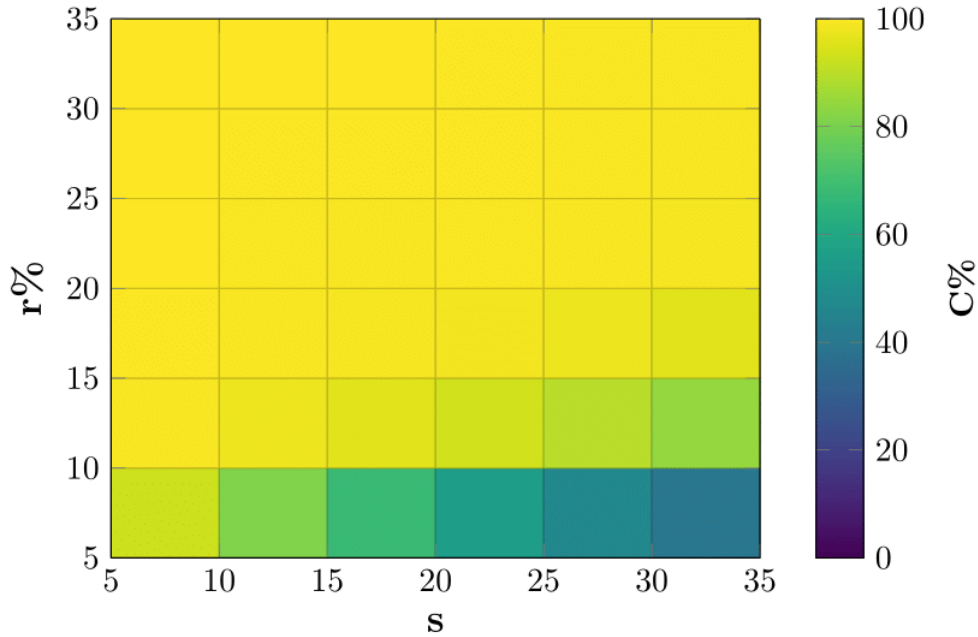


Figure 5.5: Average percentage of cooperator victories for grid size  $L = 40$  as a function of replacement rate  $r$ , and the steps per generation  $s$  using the ‘nearby’ placement mechanism.

placement strategy. In the previous two sets of experiments, a random placement strategy placed new agents in a random free cell in the environment, without regard to the location of their parent, the agents they were replacing, or to other agents in their neighbourhood. The ‘nearby’ placement strategy places new agents in the neighbourhood (i.e. the surrounding 8 cells) of their parent, if a free space exists, and otherwise places them randomly as before. This strategy is more lifelike and realistic, and it is hypothesised that this can promote cluster formation.

Figure 5.5 shows the percentages of cooperator victories across the range of evolutionary settings using the ‘nearby’ placement mechanism. Cooperation emerges in almost 100% of simulations for the vast majority of scenarios, and is only hindered in the most restrictive of evolutionary settings. Figure 5.6 shows that these results are replicated across the density levels without significant variation.

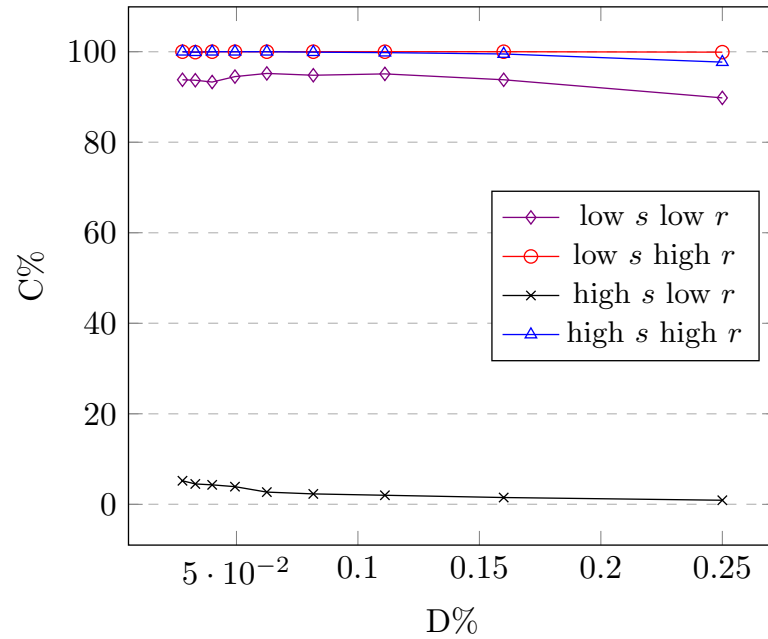


Figure 5.6: Average percentage of cooperator victories,  $C$ , for a number of evolutionary settings as a function of density,  $D$ , using ‘nearby’ placement. The low and high values are  $r = 5, 35$  and  $s = 5, 35$ .

#### 5.2.4 Genotype Consistency

In order to better understand the reasons for obtaining higher levels of cooperation in specific evolutionary settings (i.e., the replacement rate  $r$ , the number of steps in a generation  $s$ ) and the grid size  $L$  (which determines the population density), the genotypic consistency is investigated across a number of simulations using both the random and ‘nearby’ placement strategies. Considering that all simulations are randomly initialised, in all scenarios, each agent could be assigned any combination of genes with equal probability. In other words, any of the  $2^8$  genotypes could be expressed in the population.

Thus, based on the outcomes generated in previous sections, the specific evolved movement behaviours of cooperative populations under the aforementioned evolutionary settings are analysed. To achieve this, at the end of each simulation, the game strategy and the most commonly occurring genotypes that emerge in the population are recorded. The majority of

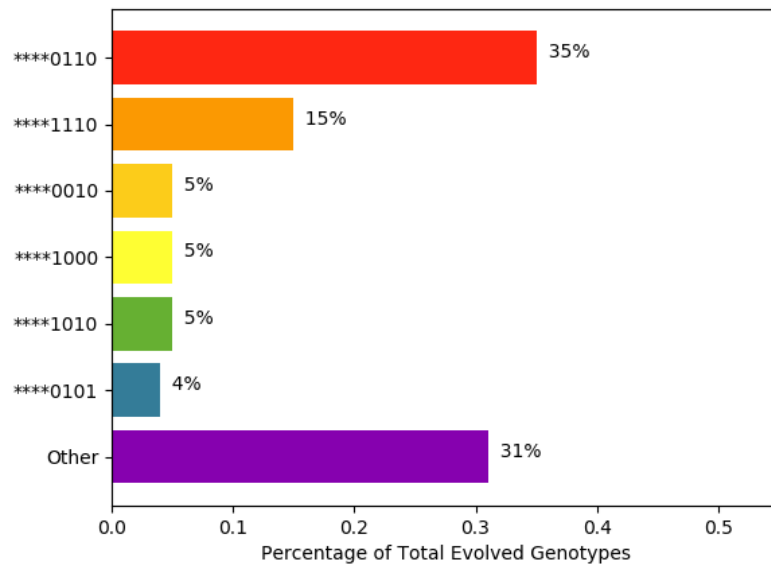
simulations will result in convergence on either cooperation or defection. A small minority of simulations will result in a ‘draw’ using evolutionary settings with a slow convergence rate. The genotypes are recorded at the end of the simulations, despite the potential for genetic drift, because the convergence point can vary depending on many different factors including initialisation, the particular evolutionary settings, population density, and which strategy is undergoing convergence.

Figure 5.7 shows the percentage break down of the most prevalent emergent genotypes (the 8-bit set of actions) from both (a) cooperator dominant and (b) defector dominant populations. It was observed that independently of the given evolutionary settings and population densities, the simulations resulting in widespread cooperation exhibit ‘01100110’ as the most commonly evolved behaviour, with the critical segment, ‘\*\*\*\*0110’, being produced in 35% of all evolved behaviours, as shown in Figure 5.7 (a). This genetic pattern corresponds to ‘follow cooperators and flee defectors’. There is a focus in the analysis on this gene section because it is critical both in terms of the evolutionary pressure it undergoes, and the major impact on the potential fitness it generates.

In other words, it is more important, from an evolutionary perspective, for an individual cooperator to move optimally in the scenario where both strategies are present than when just interacting with other cooperators. This is because there are fewer gene combinations that would lead to being punished in the latter scenario. For example, the ‘stay still’ behaviour, ‘oo\*\*\*\*\*’, in the gene segment corresponding to ‘only cooperators present’, is almost functionally equivalent to the ‘follow’ behaviour, ‘o1\*\*\*\*\*’, because both result in actions that lead to a continuation of the beneficial interactions with cooperators. However, the ‘stay still’ behaviour, ‘\*\*\*\*oo\*\*’, in the gene segment corresponding to ‘cooperators with defectors present’ is significantly worse than the ‘follow’ behaviour, ‘\*\*\*\*o1\*\*’, as it results in continual harmful interactions with defectors.

Additionally, Figure 5.7 (a) shows that the second most frequently occurring set of genotypes in simulations resulting in

(a) Evolved Cooperator Genotypes



(b) Evolved Defector Genotypes

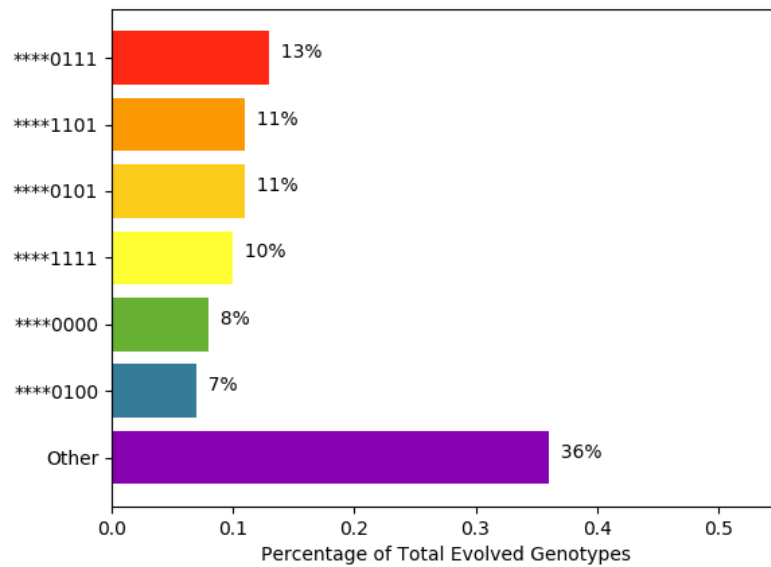


Figure 5.7: Percentage distribution of the most frequently evolved genotypes (the 8-bit set of actions) expressed in a population, using 'random' placement, where (a) cooperation dominates and (b) defection dominates. The extruded segments represent the most commonly evolved genotypes. The 'Other' segment represents the combined total of the less frequently evolved genotypes.

the emergence of cooperation is '\*\*\*\*1110'. These genotypes are genetically similar, and constitute a reasonable approximation of the 'good' solution, as they often produce actions that are phenotypically identical. For example, the 'random' behaviour, '\*\*\*\*11\*\*', in the above genotype produces the more beneficial action, 'follow', in a significant percentage of interactions.

Moreover, due to the lack of genetic mutation in the evolutionary process, once a population reaches the point of convergence, meaning that agents are no longer subject to the same level of evolutionary pressure, it may settle on a sub-optimal solution. These results indicate that the *Follow-Flee* pattern is usually the most beneficial mobility strategy for the creation of cooperative clusters which leads to the evolution of cooperation. These patterns are not found in the simulations resulting in defector dominance. As shown in Figure 5.7 (b), the *Follow-Flee* movement pattern does not appear among the most commonly evolved genotypes. It is clear that defectors are subject to much less selective pressure to optimise their mobility.

The genotypic consistency of populations using 'nearby' placement is also investigated. As shown in Figure 5.8, the results are largely similar, but one noteworthy deviation is that the *Follow-Flee* movement pattern is less pronounced. It's clear that cooperators are under less selective pressure to evolve 'good' contingent mobility strategies.

### 5.2.5 *N-Player Interaction Model*

The impact of the N-Player interaction model on the co-evolution of cooperation and mobility using the extended mobility model is also considered, as in Chapter 4. Using the same experimental set up as in previous sections, the evolutionary settings, environment density, and the interaction model are varied to investigate the differences, if any, that arise using the extended mobility model in the N-Player PD.

Figure 5.9 and 5.10 show results of simulations with varying the evolutionary settings and population density in the N-

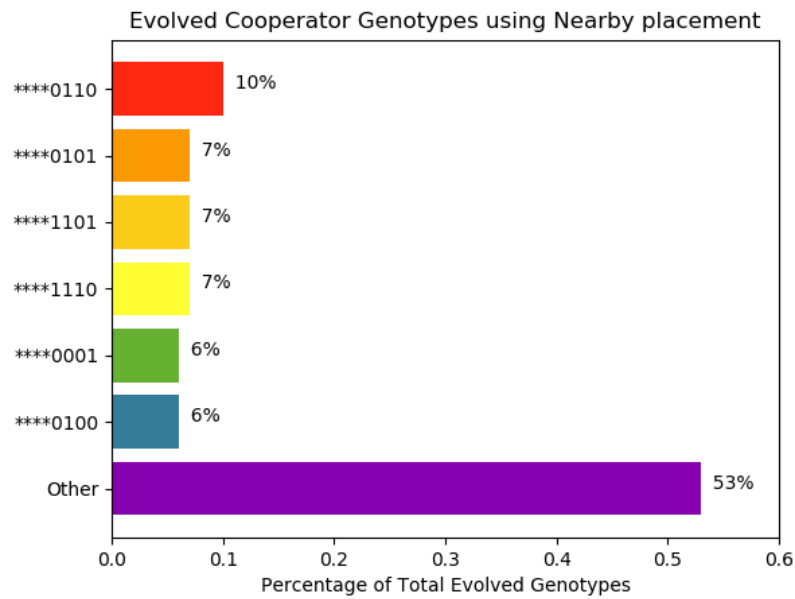


Figure 5.8: Percentage distribution of the most frequently evolved genotypes (the 8-bit set of actions) expressed in a population where cooperation dominates using ‘nearby’ placement. The extruded segments represent the most commonly evolved genotypes. The ‘Other’ segment represents the combined total of the less frequently evolved genes.

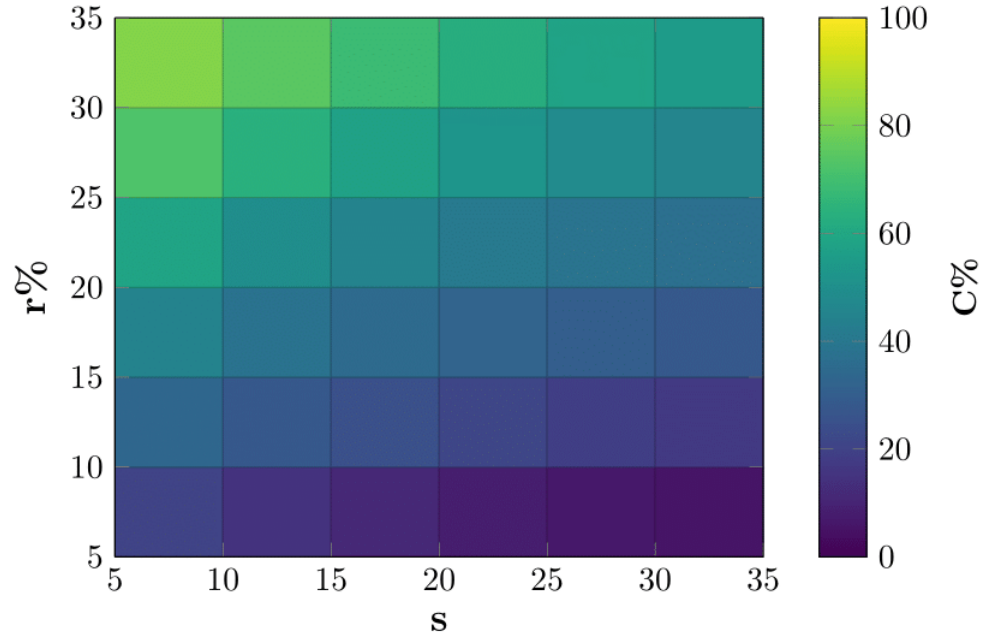


Figure 5.9: Average percentage of cooperator victories in the N-Player PD for grid size  $L = 40$  as a function of replacement rate  $r$ , and the steps per generation  $s$  using the ‘random’ placement mechanism.

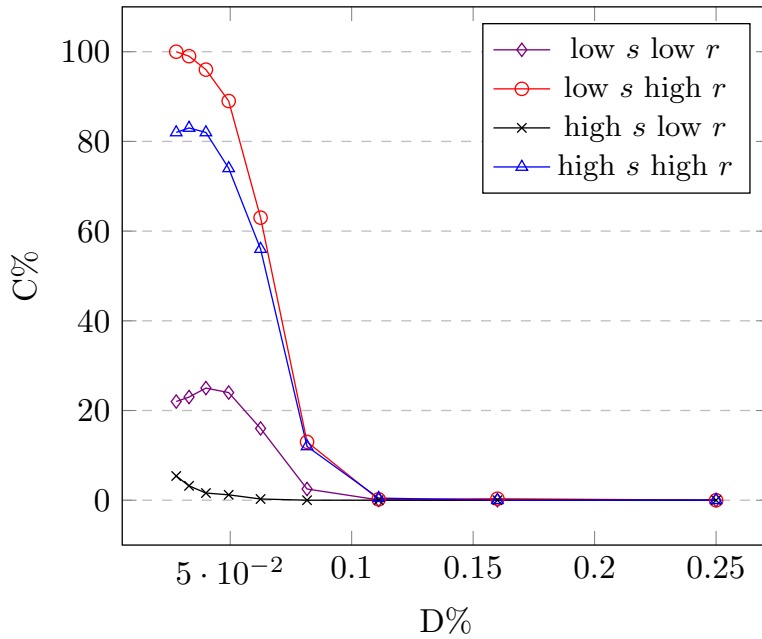


Figure 5.10: Average percentage of cooperators victories,  $C$ , in the N-Player PD for a number of evolutionary settings as a function of density,  $D$ , using ‘random’ placement. The low and high values are  $r = 5, 35$  and  $s = 5, 35$ .

Player interaction model using the ‘random’ placement mechanism. These experiments reiterate the result that it is significantly harder to promote the evolution of cooperation using the N-Player interaction model, as described in Chapter 4. In Figure 5.9, similar to previous results using the N-Player model, short generations and high replacement rates facilitate the greatest levels of widespread cooperation. However, cooperation emerges at this moderate density at much lower rates than in simulations using the 2-Player interaction model. Furthermore, Figure 5.10 illustrates that cooperation emerges at similar levels in low density settings but cannot emerge in a wide range of moderate to high density levels, in which cooperation does emerge using the 2-Player model.

Figure 5.11 and 5.12 show results of simulations varying the evolutionary settings and population density in the N-Player interaction model using the ‘nearby’ placement mechanism. It is clear that overall the evolution of cooperation in these scenarios is severely hindered using ‘nearby’ placement. This is the case both when the evolutionary settings and density is

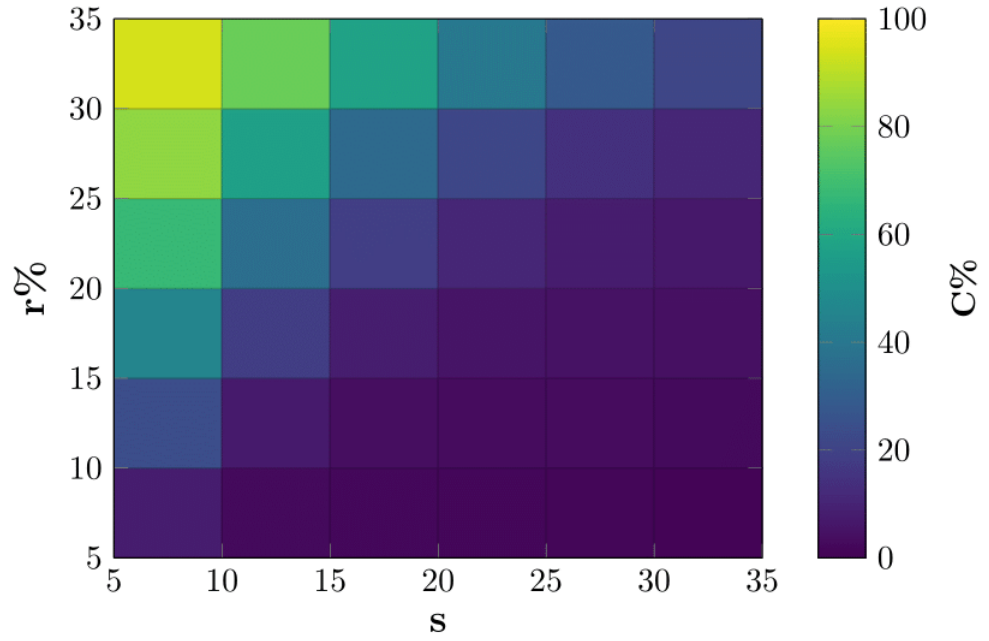


Figure 5.11: Average percentage of cooperator victories in the N-Player PD for grid size  $L = 40$  as a function of replacement rate  $r$ , and the steps per generation  $s$  using the ‘nearby’ placement mechanism.

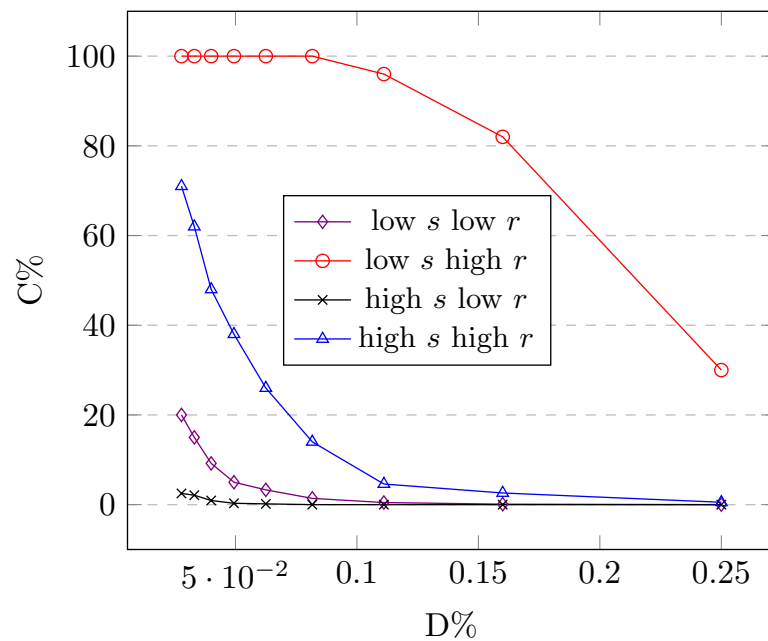


Figure 5.12: Average percentage of cooperators victories,  $C$ , in the N-Player PD for a number of evolutionary settings as a function of density,  $D$ , using ‘nearby’ placement. The low and high values are  $r = 5, 35$  and  $s = 5, 35$ .

varied. In fact, there is a wide range of densities where cooperation simply cannot emerge. However, in a small minority of combinations of evolutionary settings, specifically with short generations and high replacement rates, cooperation emerges in higher levels compared to those produced with 'random' placement by the N-Player model. Furthermore, cooperation emerges in comparable levels to those produced with 'nearby' placement by the 2-Player model. As was the case with 'nearby' placement in the 2-Player model, agents are under less selective pressure to produce optimal mobility strategies. As a result, cooperator clusters are more vulnerable to the increased exploitative influence of the defectors in the more difficult game, thus defection emerges more readily.

Figure 5.13 shows the percentage break down of the most prevalent emergent genotypes from cooperator dominant simulations using both the (a) 'nearby' placement and (b) 'random' placement mechanisms. As in the 2-Player interaction model, the *Follow-Flee* genetic pattern emerges as the most commonly evolved behaviour. However, it was observed that the critical segment of the evolved genotypes in the N-Player version of the game instead corresponds to the scenarios where single strategies are present, ie '0110\*\*\*\*'. Using the N-Player interaction model cooperation only emerges in low density environments where agents have fewer encounters on average, therefore there is less selective pressure on the gene segments describing scenarios with both cooperators and defectors.

As in the 2-Player model, the populations evolved using the 'nearby' placement mechanism are under less selective pressure to be optimally mobile than those evolved using 'random' placement. However, unlike in the 2-Player model where the cooperator clusters created by the 'nearby' placement mechanism propelled the evolution of cooperation across the board, in the N-Player model 'nearby' placement actively hinders the spread of cooperation. Due to the lack of evolutionary pressure, the clusters created by 'nearby' placement most often do not contain the necessary genotypes to avoid repeated interactions with defectors. This wasn't an issue in the 2-Player game

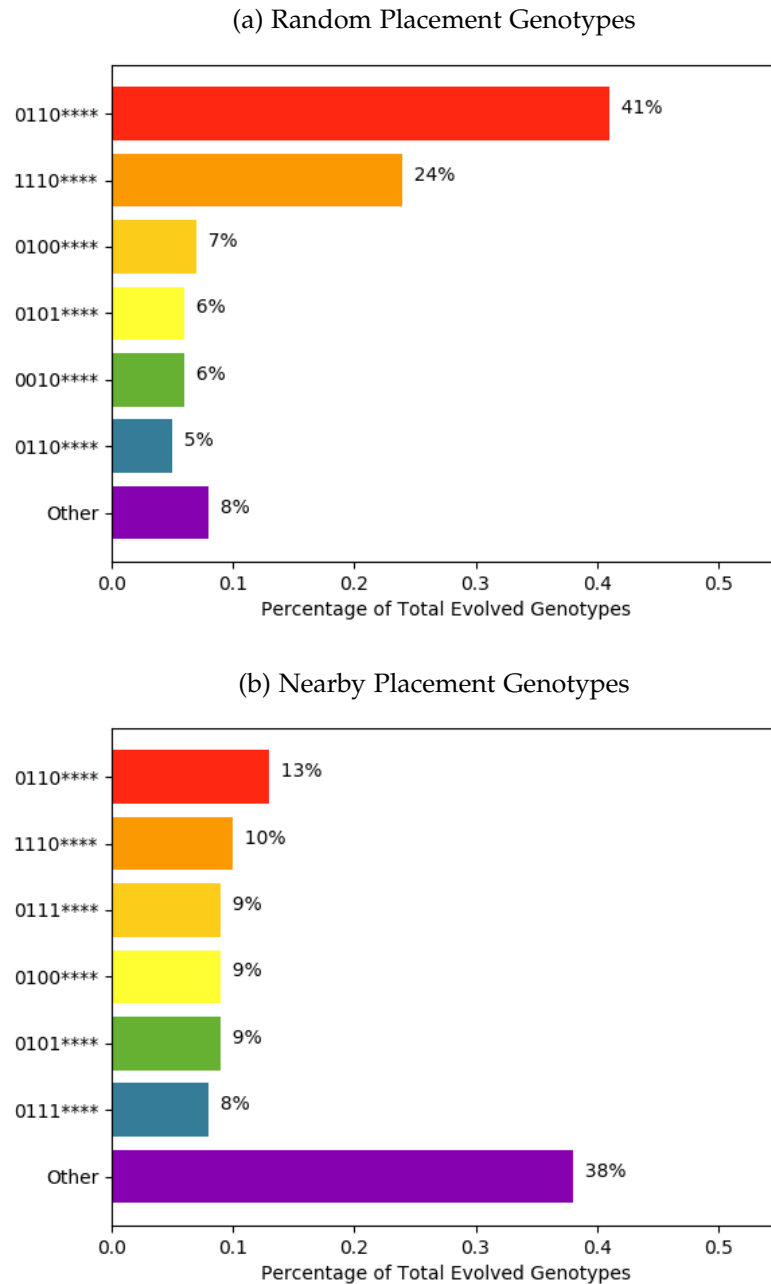


Figure 5.13: Percentage distribution of the most frequently evolved genotypes expressed in a population where cooperation dominates in the N-Player PD using (a) 'random' placement and (b) 'nearby' placement. The extruded segments represent the most commonly evolved genotypes. The 'Other' segment represents the combined total of the less frequently evolved genotypes.

because the cooperators in clusters were able to accumulate enough fitness to overcome the exploitation by the defectors without the need to flee from them. However in the N-Player game defectors have a much larger exploitive influence and it becomes much more important to have contingent mobility strategies.

### 5.3 DISCUSSION

A number of environmental and evolutionary factors governing the emergence of cooperation within populations of mobile agents have been observed in both the 2-Player and N-Player PD. Population density, agent lifespan, the choice of placement strategy, and the interaction model all distinctly impact the formation of cooperator clusters, which is the most critical factor in the evolution of cooperation among agents using pure strategies. These clusters emerge as a consequence of the agents' evolved mobile strategies.

Using the standard 2-Player PD, agent lifespans consisting of short generations and high replacement rates favour, and often promote, the evolution of cooperation. These evolutionary settings curb the 'free rider' effect once the cooperators have learned good movement strategies, which form clusters, allowing agents to avoid repeated exploitation by defectors. If, in the initial generations of a simulation, cooperators have not learned to cluster by following neighbouring cooperators and fleeing from neighbouring defectors, defection will emerge. The *Follow-Flee* contingent mobility strategy is critical for cooperators to evolve in order for them to be successful in a simulation. The results in this chapter show that in every simulation in which cooperation emerges some approximation of *Follow-Flee* is evolved.

Population density has a major impact on the emergence of cooperation in spatial environments with a mobile population, because it directly impacts the interaction rate with defectors. Cooperation is most likely to emerge when clusters of coopera-

tors, with appropriate genes, are formed and allowed to grow unimpeded in the initial timesteps of a simulation. The chance of this occurring is significantly higher in sparse environments. In dense environments, cooperators have a higher chance of being exploited by defectors, as a result neither the evolved movement strategies nor the evolutionary settings can ignite the evolution of cooperation, unless the initial conditions are particularly favourable. On the other hand, a sparse environment almost guarantees the emergence of cooperation. Clusters in sparse environments have a higher chance of avoiding exploitation, thus allowing its members to learn beneficial movement patterns, and obtain a high fitness score.

There is a clear interplay between the population density and the evolutionary settings in this work. In general, the more time and space agents have to learn ‘good’ movement strategies, the more likely cooperation is to emerge. It is even possible to construct a set of parameters to ensure that cooperation emerges in the vast majority of simulations. However, unsympathetic agent lifespans will result in total defector domination, regardless of the population density. These harsh evolutionary settings favour defectors because the movement strategies, which give cooperators the competitive edge over the defectors, are not learned in sufficient time to be effective.

The scenarios discussed thus far assume the use of the ‘random’ placement strategy, however it is observed that a substantial decline in influence of both agent lifespan and population density occurs when the ‘nearby’ placement strategy is in effect. In fact, these factors become almost irrelevant (see Figure 5.5 and 5.6). Furthermore, the *Follow-Flee* genotype doesn’t occur with the same frequency in evolved cooperator populations as with the ‘random’ placement strategy. This is due to there being significantly less pressure to learn clustering behaviours as ‘nearby’ placement ensures, where possible, that clusters grow.

Using the N-Player PD, this extended mobility model produced results similar to those recorded in Chapter 4. Cooperation was generally harder to promote using the interaction model that gave greater influence to ‘free riding’ defectors. How-

ever, in one particular set of experiments the results wildly disagreed. The ‘nearby’ placement mechanism in simulation using the 2-Player interaction model almost guarantees the total adoption of cooperation by a population in the majority of evolutionary settings. The opposite is true for this placement strategy in simulations using the N-Player interaction model. As already discussed, ‘nearby’ placement reduces the evolutionary pressure on a population to learn clustering mobility strategies by creating those same cooperator clusters. This is not sufficient for agents playing the N-Player game, especially when the extended model has increased the strategy space making ‘good’ mobility strategies harder to evolve. In the N-Player environment it is not sufficient for agents to simply form clusters they must also form clusters that have the ability to evade defectors.

#### 5.4 CONCLUSION

In this chapter, a novel mobility model was presented, in which agents pro-actively seek out better locations by moving locally in response to their highly dynamic environment. The influence of several environmental and evolutionary factors on the emergence of cooperation was demonstrated among mobile agents using this model. Contingent mobility strategies were evolved, which closely resemble *Follow-Flee*, that are extremely successful at spreading cooperation throughout a mobile population without the need for complex computation, costly memories, or central control.

It is shown that appropriately tuning the evolutionary process in conjunction with a favourable population density and sufficiently mobile agents can almost guarantee cooperation to emerge from a randomly initialized population. Additionally, results have been presented indicating that cooperation may emerge in a population with sub-optimal movement patterns given a placement strategy that enhances cooperator clusters.

It has been shown that the impact of certain environmental and evolutionary settings can substantially diminish oth-

ers. The 'nearby' placement strategy creates cooperator clusters with such efficiency that the agents are under considerably less pressure to evolve the clustering behaviours.

Finally, the extensions made to the mobility model have been shown to produce higher levels of cooperation in both the 2-Player and N-Player variants of the PD. Cooperation can be guaranteed to emerge in a range of environmental and evolutionary settings using an unseeded population.

## CONCLUSION

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This chapter provides a summary of the answers from the previous chapters to each of the research questions and hypotheses outlined in Chapter 1. In this chapter, the research questions are first restated and answered. Each of the hypotheses are examined and are discussed in terms of the extent to which they have been shown to be true or false by the work in this thesis. Finally, potential future avenues of work stemming from this research are discussed.

### 6.1 SUMMARY

Chapter 1 introduces the key concepts and domains central to the research conducted for this thesis. Game theory and its ubiquitous game, the Prisoner's Dilemma, are outlined in terms of the apparent contradictions they highlight between rational and altruistic choices. The evolution of cooperation is introduced alongside the various mechanisms and techniques which have been traditionally used to simulate this emergent phenomenon. Finally, mobility is introduced and discussed in terms of its potential to more accurately describe the outbreak of cooperation between simple agents in dynamic environments. In addition, the specific brand of contingent mobility proposed in this work is outlined and motivated.

Chapter 2 further discusses the concepts dealt with in this thesis in more depth, and examines the related work conducted in the various research areas.

Chapter 3 introduces the contingent mobility strategy *Follow-Flee* central to the work in this thesis. It is compared to a prominent strategy in the literature, *Walk Away*, in terms of its ability to promote the evolution of cooperation. *Follow-Flee* is shown

to generate higher levels of cooperation in a range of environments and evolutionary settings. Additionally, it is shown that *Follow-Flee* can invade *Walk Away* when they compete directly, despite the fact that both strategies are mutually cooperative.

Chapter 4 introduces an evolutionary model in which agents co-evolve their game strategies and their mobility strategy. Both the 2-Player and N-Player variants of the PD were considered and examined in terms of their ability to promote the evolution cooperation and the contingent mobility strategies they each produced. This chapter also introduces a number of cluster measures to both quantitatively and qualitatively analyse cooperator clusters over the course of a simulation. The formation of cooperator clusters is shown to be pivotal in the evolution of cooperation. Simulations that result in widespread cooperation produce both large, cohesive clusters and mobility strategies that enable clustering.

Chapter 5 investigates the co-evolution of cooperation and mobility using a second, more expressive, mobility model in a wide range of environments and evolutionary settings. Contingently mobile strategies, and particularly *Follow-Flee*, are shown to be essential to the evolution of cooperation because of their ability to create and sustain cooperator clusters. Additionally, it is shown it is possible to construct a combination of evolutionary settings in which cooperation is guaranteed to evolve alongside suitable mobility strategies. In fact, the only scenario in which cooperation emerges without the presence of these contingent mobility strategies is when the evolutionary process itself promotes the formation of cooperator clusters.

## 6.2 ANSWERS TO RESEARCH QUESTIONS

In Section 1.1, four research question were proposed. This section provides an answer to each of these questions based on conclusions from previous chapters.

**Research Question 1:** *Does contingent mobility promote the evolution of cooperation in the Prisoner's Dilemma game with highly dynamic environments due to a highly mobile population of agents?*

In Section 3.1, a common methodology is outlined, which describes the environment topology, agent representation, and the evolutionary process used, with noted exceptions, throughout the thesis. In this work, during a simulation the population is in constant motion, in this way the agents are highly mobile. In other work covered in Chapter 2, mobility is conditional; the movements of agents are based on environmental states or determined by mobility rates.

At every time-step each agent has the opportunity to move according to their strategy. Cooperation is readily promoted by these highly mobile agents using 'good' mobility strategies as demonstrated in Chapters 3, 4, and 5.

**Research Question 2:** *Can a contingent mobility strategy, capable of promoting the evolution of cooperation, be constructed using only local environmental information with limited agent complexity?*

The common methodology, outlined in Section 3.1, also describes the capabilities of the agents developed in this thesis; this includes their range of motion, their perception, and their knowledge requirements. Agents can only consider the information they can obtain in any given turn, which is limited to the strategies and positions of their neighbours within the Moore neighbourhood. They calculate the best position to move to from the available positions in that region of the lattice. Agent memories do not persist over time, and they use pure strategies for playing the PD game. The agents in this work don't rely on global information, such as influence or reputation as seen in the literature, and they don't employ any 'look-ahead' functions to test far away locations.

Chapter 3 presents the proactive contingent mobility strategy, *Follow-Flee*, which is designed to function within these minimal memory requirements and knowledge restrictions. This strat-

egy instructs its agents to follow neighbouring cooperators and to flee from neighbouring defectors, and has been shown to promote the evolution of cooperation.

**Research Question 3:** *Can mobility be co-evolved with cooperation in order to produce an uncomplicated contingent mobility strategy capable of promoting cooperation in dynamic environments?*

In both Chapter 4 and 5, the strategy with which agents play the game, cooperate or defect, and the agents' mobility strategy are co-evolved. Agent fitness is measured using the cumulative payoff received from playing the game, after which high scoring agents replicate, and low scoring agents die out; the frequency of these processes is also then varied. Cooperation and contingent mobility strategies are shown to co-evolve in a range of evolutionary settings.

In Chapter 5, a more expressive mobility model, which considers a wider range of movement action is designed and adopted. In both Chapter 4 and 5, it was demonstrated that cooperation emerged in a population when mobility strategies that approximated *Follow-Flee* were evolved. So long as cooperators developed a mobility strategy that continued cooperator interactions and discontinued defector interactions, cooperation had a reasonable chance to emerge through the formation of cooperator clusters.

Additionally, it was demonstrated that cooperation did not emerge when contingent mobility strategies did not evolve. Defection spreads and dominates a population if contingently mobile strategies, such as *Follow-Flee*, do not appear within the first generations. The cooperators need the advantage of consistently interacting in clusters in order to overcome the 'free rider' effect caused by highly mobile defectors.

**Research Question 4:** *Can contingent mobility promote the evolution of cooperation in the N-Player extension of the Prisoner's Dilemma game?*

Two interaction models were used in this work; the standard 2-Player interaction model for the PD and the extended N-Player model. In the 2-Player model, agents play a single round of the game with each of their neighbours individually. In the N-Player model, agents play a single combined game with their neighbours. Many versions of this type of social dilemma exist and could have been considered, but in this work, a version of the N-Player PD that could be directly compared to the 2-Player interaction model was chosen. In these scenarios, individual defectors have a greater exploitative effect on cooperators. In a large game defectors earn a higher relative payoff than their 2-Player counterparts playing multiple games. Free riding defectors have a significant impact on the evolutionary process by consistently obtaining high payoff.

In Chapter 4, cooperation is shown to be promoted using contingent mobility strategies even using the N-Player version of the game. Additionally, in low density environments where cooperator clusters have a higher likelihood of forming, aided by contingent mobility strategies to avoid repeated defector interactions, cooperation is promoted to a greater extent in the N-Player interaction model than in the standard 2-Player version. This phenomenon is also observed in Chapter 5 using a more expressive mobility model. The larger N-Player games result in a higher combined payoff for cooperators in unexploited clusters allowing cooperation to proliferate more readily across generations. Both interaction models show a similar pattern in terms of their ability to spread cooperation among mobile agents in dilute environments, with the N-Player model exhibiting more polarised results.

### 6.3 HYPOTHESES

In this section, an answer is provided for each hypothesis put forward in Section 1.4.

**Hypothesis 1:** *Follow-Flee is a viable competitive strategy for promoting cooperation in the spatial Prisoner's Dilemma game.*

In Chapter 3, *Follow-Flee* is shown to out perform *Walk Away*, a prominent contingent mobility strategy from the literature. It was demonstrated that *Follow-Flee* could dominate and invade the other strategy, despite the fact that they are both mutually cooperative strategies. In a static environment, two strategies that always cooperate, and have no other competing elements, would largely co-exist. Additionally, in Chapter 4 and 5, it was shown that *Follow-Flee*, and strategies closely resembling it, were the contingent mobility strategies most consistently evolved by populations who also evolved to cooperate.

**Hypothesis 2:** *Contingent mobility promotes the creation of cooperative clusters of agents playing the Prisoner's Dilemma using pure strategies in a range of environmental and evolutionary settings.*

In Chapter 4, a cooperator cluster is described as a set of two or more spatially adjacent agents, who 'cooperate' in the PD, which maintain this spatial connection over some period of time. A set of cluster metrics are presented to record and analyse the growth of clusters over the course of a simulation.

In this work, the size and quality of clusters were observed to increased over time in populations that evolved contingent mobility strategies. The *Follow-Flee* mobility strategy, proposed in this Chapter 3 and emergent in Chapter 4 and 5, actively maintains these clusters by favouring movements which continue cooperator connections. Additionally, in Chapter 5, the formation and spread of cooperator clusters by agents using contingent mobility strategies is observed visually.

**Hypothesis 3:** *The formation and growth of cooperative clusters promotes the evolution of cooperation among agents using pure strategies in a range of environmental and evolutionary settings.*

In simulations that result in the evolution of cooperation the population has also evolved mobility strategies that promote the creation of cooperator clusters. In addition, in simulations that result in widespread defection, the population shows little preference for learning particular movement strategies, and they ones that do emerge don't create cooperator clusters. Simulation that don't exhibit cluster formation do not promote the evolution of cooperation.

In each of the results chapters, contingent mobilities are shown to promote the evolution of cooperation in the PD through the creation and expansion of cooperator clusters that can avoid repeated interactions with defectors. Additionally, it is shown that where contingent mobility strategies are not sufficiently present in a population, defection emerges.

**Hypothesis 4:** *The evolution of cooperation in mobile populations on spatial environments is heavily influenced by a range of environmental and evolutionary settings, including: population density, agent lifespan, and the birth-death update rules.*

Density has an enormous impact on mobile populations, and as a result we examine its effects specifically in each of the three results Chapters 3, 4 and 5. In order for a population to be mobile in a spatial environment, specifically a toroidal lattice in this work, the environment must be dilute, i. e. have gaps or free spaces. Density directly impacts the number of interactions individual agents have over their lifespan. A fully occupied lattice grid and environments with high density containing agents using pure strategies will greatly favour the emergence of defection. Cooperators gain their advantage over defectors by forming clusters, but with little to no space to move, the formation of these clusters is countered by the 'free rider' effect. A single defector may be able to exploit a large number of cooperators

without repercussion. As grid density decreases, the likelihood of encountering a defector in turn decreases, which increases the chances that an isolated pair of cooperators may form a cluster and have the time and space to grow. By learning mobility strategy strategies, cooperators further increase the probability of forming cooperator clusters, and avoid future interactions with defectors.

A number of evolutionary settings were examined and varied during the course of this research, and are examined in Chapters 3 and 5. They include generation length, which denotes the number of turns an agent takes each generation, and the replacement rate, which determines the percentage of agents that die and reproduce at the end of each generation. Long generation lengths allow mobile defectors time to generate higher fitness leading to a runaway 'free rider' effect. While short generation lengths allow cooperators to generate higher levels of fitness, by forming fortified clusters, before the 'free rider' effect takes over. High replacement rates favour the evolution of cooperation. They negate the effect of free riding defectors dominating the evolutionary process by allowing clustered cooperators to proliferate. Additionally, the evolution of cooperation is highly dependent on agents learning good mobility strategies, which improve the quality of these clusters in the next generation. The replacement rate has an overall greater positive impact on the evolution of cooperation than generation length. However, both should be tuned in tandem in order to produce the highest levels of cooperation.

Two placement mechanisms within the birth-death process for agent offspring were considered. In the evolutionary process, throughout this work, newly created agents are placed randomly back into the environment. In Chapter 5, changing this mechanism is investigated by instead placing newborn agents in the neighbourhood of their parent. In any scenario where this would not be possible, agents revert to using the original random placement mechanism. The nearby placement mechanism is more successful at promoting the evolution of cooperation. It grows cooperator clusters by placing new cooperators

next to their parent, whom have already proven to perform well. Surprisingly, agents in these simulations are put under less evolutionary pressure to produce optimal mobility strategies. The critical purpose of learning these mobility strategies is the creation and expansion of cooperator clusters, here this phenomenon is achieved by this placement mechanism.

#### 6.4 FUTURE WORK

There are a number of potential future avenues of research have been identified stemming from this thesis.

In the short term, the agent representation could be modified to allow for the inclusion of noise in the model. A noise variable could be introduced, which would cause agents to incorrectly identify interactions with their neighbours for some percentage of interactions. This would enable a further analyse the robustness of the evolved mobile strategies, including *Follow-Flee*, that arose from this research.

Another avenue of immediate investigation would be to consider the influence of contingent mobility in other social dilemma games. In the literature there is less focus on mobility in games such as the Stag Hunt, the Snow Drift game, or the Ultimatum game. Additionally, other versions of the N-Player PD or similar resource allocation problems could be investigated using this evolutionary mobility model.

In the medium term, the mobility model could be expanded to consider the impact on the evolution of cooperation of agent teleportation, i.e. agents moving to a location outside their neighbourhood, within their own lifespan. This ability would incur a cost to their fitness and allow them to randomly, or deterministically, jump to a distant location on the grid. This ability could be considered as an exploratory action available to agents in unfavourable neighbourhoods. Currently, both placement mechanisms allows for some amount of random relocation, however this only affects newborn agents.

Alternatively, other types of network topology could be considered to evaluate the proposed model in a range of more realistic situations and environments. Some of the most prominently used topology in this domain include small world graphs, scale-free graphs, and social networks.

Finally in the long term, it may be possible to consider grounding these models in physically embodied agents using simple robots. As the fields of robotics and evolutionary robotics continue to expand a need may arise for coordinating individual and collectively rational behaviours among physical agents. The brand of contingent mobility proposed in this thesis, *Follow-Flee* in particular, could be adapted to meet the strict requirements of physical robots, i.e. limited memory, process power, visibility, and range of mobility, all constraints that were considered when this contingent mobility model was designed.

## 6.5 CONCLUDING REMARKS

In conclusion, this thesis has shown that cooperation can be promoted in the Prisoner's Dilemma using contingent mobility strategies provided that they facilitate the growth of cooperator clusters. *Follow-Flee*, a contingent mobility strategy evolved by populations of highly mobile agents of limited complexity, has been shown to be a viable competitive strategy for both the 2-Player and N-Player versions of the game. Finally, a number of avenues for future work have been specified.

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